The Rotational and Gravitational Signature of Recent Great Earthquakes

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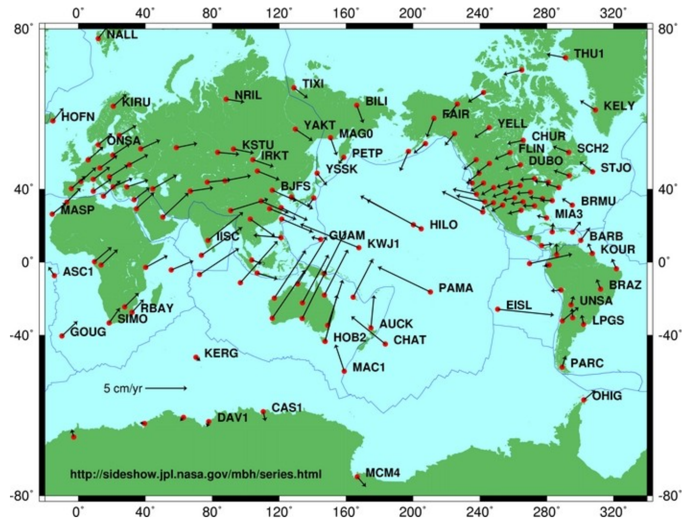
7th IVS General Meeting

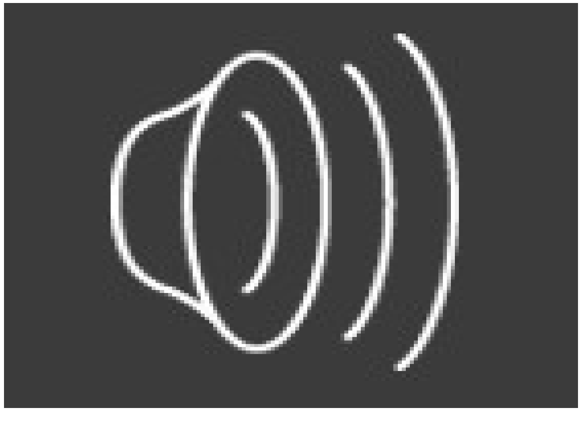
March 4–9, 2012
Madrid, Spain

The Three Pillars of Geodesy

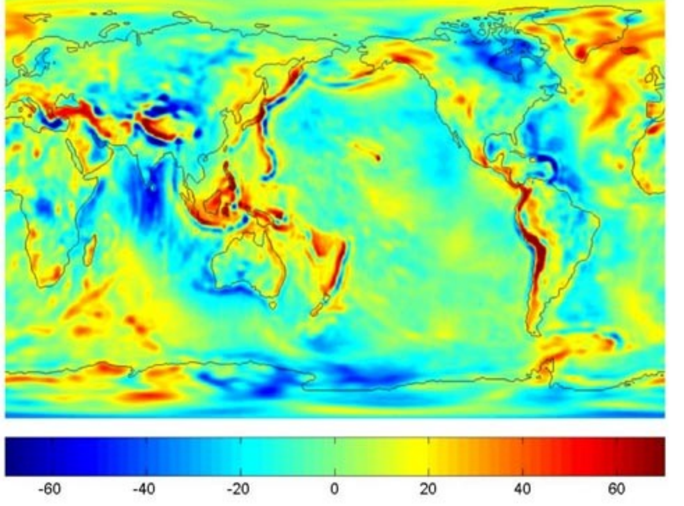


Shape & Deformation

Earth Rotation



Gravity & Geoid



Introduction

- The three pillars of geodesy
  - Geometric shape, rotation, and gravity
    - Static and time varying
- Are related by common observing systems
  - Examples: SLR, GPS, DORIS, VLBI (shape and rotation)
- Are related by common sources of excitation
  - Examples: atmosphere, oceans, hydrology, earthquakes, GIA
- But are sometimes modeled separately
  - Example: flat Earth models for earthquake displacements, spherical Earth models for rotation
- A unified modeling approach
  - Allows excitation process to be studied using all geodetic data
  - Example: earthquakes

Geodetic Effects of Earthquakes

- Flat Earth models
  - Commonly used to model site displacements
  - May or may not include effects of layering
  - Do not include effects of sphericity
    - Important for great earthquakes having rupture lengths of 1000 km (10°) or more
- Spherical Earth models
  - Commonly used for rotation effects
  - Often used for gravity effects
- A unified mode sum approach
  - Based on a realistic Earth model (PREM)
  - Automatically accounts for effects of sphericity, layering, self-gravitation

Site Displacements

- Equation of motion

$$\nabla \cdot \tau + \mathbf{f_g} + \mathbf{f_s} = \rho(\mathbf{r}) \frac{\partial^2 \mathbf{u}}{\partial t^2}$$

- Solve by expanding displacement field

$$\mathbf{u}(\mathbf{r},t) = \sum_k a_k(t)\, \mathbf{u}_k^*(\mathbf{r})$$

  - Normal mode eigenfunctions

$$\mathbf{u}_k(\mathbf{r}) = {}_nU_l(r)\, Y_{lm}(\theta,\lambda)\, \hat{\mathbf{r}} + {}_nV_l(r) \frac{\partial Y_{lm}}{\partial \theta}\, \hat{\boldsymbol{\theta}} + {}_nV_l(r) \frac{1}{\sin\theta} \frac{\partial Y_{lm}}{\partial \lambda}\, \hat{\boldsymbol{\lambda}}$$

  - Expansion coefficients (static limit)

$$a_k(\infty) = \frac{M_o}{\omega_k^2}\, \hat{\mathbf{M}} : \boldsymbol{\varepsilon}_k(\mathbf{r_s})$$

- Normal mode eigenfrequencies and eigenfunctions
  - Computed using MINOS program provided by Guy Masters

Earth Rotation

- Conservation of angular momentum
  - Earthquakes are internal to the Earth (no load Love numbers)
  - Earthquakes occur suddenly (motion effects neglected)
- Length of day

$$\frac{\Delta\Lambda(t)}{\Lambda(t)} = \frac{\Delta I_{zz}(t)}{I_{zz}^{(m)}}$$

- Polar motion excitation

$$\Delta\chi_x(t) + i\,\Delta\chi_y(t) = \frac{1.61}{I_{zz} - I_{xx}}\left[\Delta I_{xz}(t) + i\,\Delta I_{yz}(t)\right]$$

- Inertia tensor

$$\mathbf{I} = \int \rho(\mathbf{r})\left[\left(\mathbf{r}\cdot\mathbf{r}\right)\mathbf{I} - \mathbf{r}\,\mathbf{r}\right]dV$$

- Perturbed inertia tensor $(\mathbf{r} \;\Rightarrow\; \mathbf{r} + \mathbf{u})$

$$\Delta\mathbf{I} = \int \rho_o(\mathbf{r})\left[2\left(\mathbf{r}\cdot\mathbf{u}\right)\mathbf{I} - \left(\mathbf{u}\,\mathbf{r} + \mathbf{r}\,\mathbf{u}\right)\right]dV$$

Gravitational Field

- Gravitational potential of Earth

$$U(\mathbf{r}) = \frac{GM}{r}\sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^{l}\left[C_{lm}\cos m\lambda + S_{lm}\sin m\lambda\right]\tilde{P}_{lm}(\cos\theta)$$

- Stokes coefficients

$$C_{lm} + i\,S_{lm} = \frac{N_{lm}}{Ma^{l}}\int_{V_o} r^{l}\,Y_{lm}(\theta,\lambda)\,\rho(\mathbf{r})\,dV$$

- Perturbed Stokes coefficients $(\mathbf{r} \;=>\; \mathbf{r} + \mathbf{u})$

$$\Delta C_{lm} + i\,\Delta S_{lm} = \frac{N_{lm}}{Ma^{l}}\int_{V_o} r^{l-1}\,\mathbf{u}\cdot(\hat{\mathbf{r}}\,l + \nabla_h)\,Y_{lm}(\theta,\lambda)\,\rho(\mathbf{r})\,dV$$

Earth Rotation & Gravitational Field

- Related via the inertia tensor
  - Elements of inertia tensor are related to the degree-2 Stokes coefficients

$$I_{xz} = -\sqrt{5/3}\,Ma^2\,C_{21}$$

$$I_{yz} = -\sqrt{5/3}\,Ma^2\,S_{21}$$

$$I_{zz} = \frac{1}{3}\left[T - \sqrt{20}\,Ma^2\,C_{20}\right]$$

- Trace of inertia tensor

$$T = 2\int \rho(\mathbf{r})\,r^2\,dV$$

- Perturbed trace of inertia tensor $(\mathbf{r} \Rightarrow \mathbf{r} + \mathbf{u})$

$$\Delta T = 4\int \rho_o(\mathbf{r})\,\mathbf{r}\cdot\mathbf{u}\,dV$$

Polar Motion Excitation

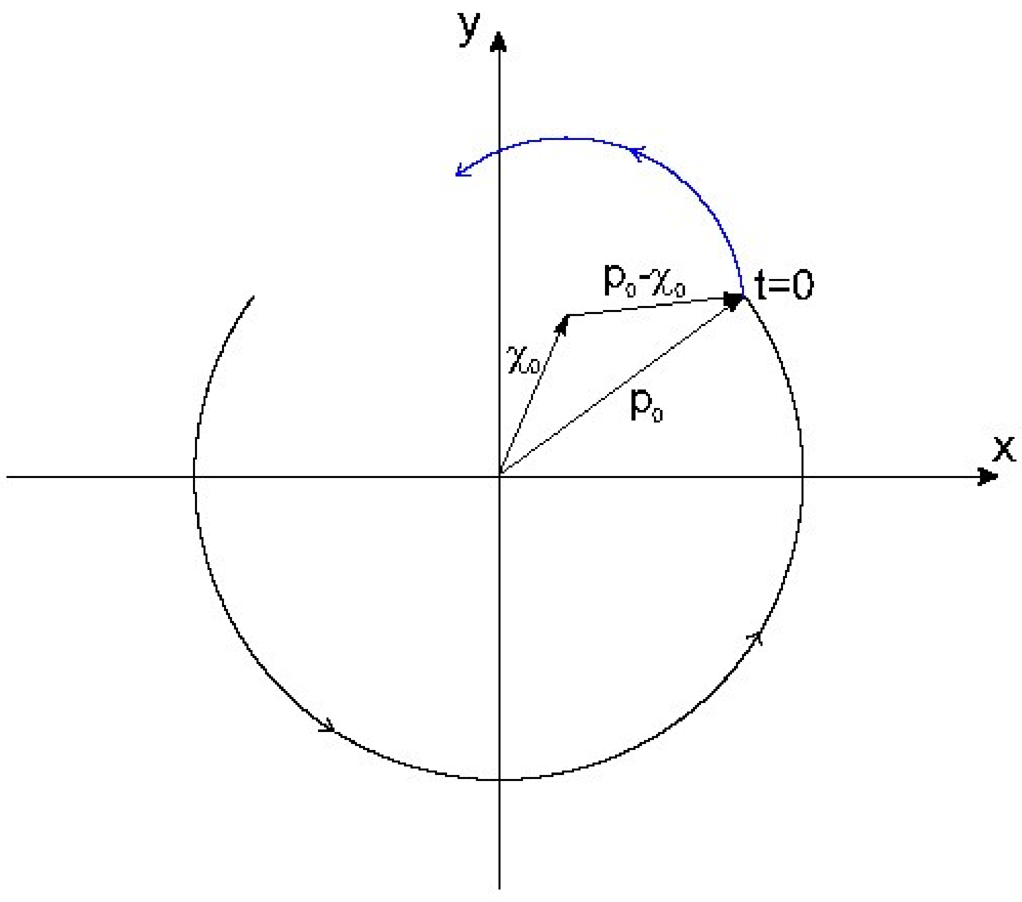


Figure 2: Excitation of polar motion by the step function.

(Brzezinski, 2005)

Recent Great Earthquakes

- 2004 Sumatra ($M_w = 9.3$)
  - 5 sub-event model of *Tsai et al.* (2005)
    - Based on seismic data
- 2005 North Sumatra ($M_w = 8.5$)
  - Caltech's Tectonics Observatory slip model with 295 patches
    - Based on GPS, seismic, and coral uplift and subsidence data
- 2007 South Sumatra ($M_w = 8.4$)
  - Caltech's Tectonics Observatory slip model with 203 patches
    - Based on GPS, seismic, and InSAR data
- 2010 Chile ($M_w = 8.8$)
  - Updated USGS slip model with 166 fault patches slipping
    - Based on seismic data
- 2011 Japan ($M_w = 9.0$)
  - Updated USGS slip model with 319 fault patches slipping
    - Based on seismic data

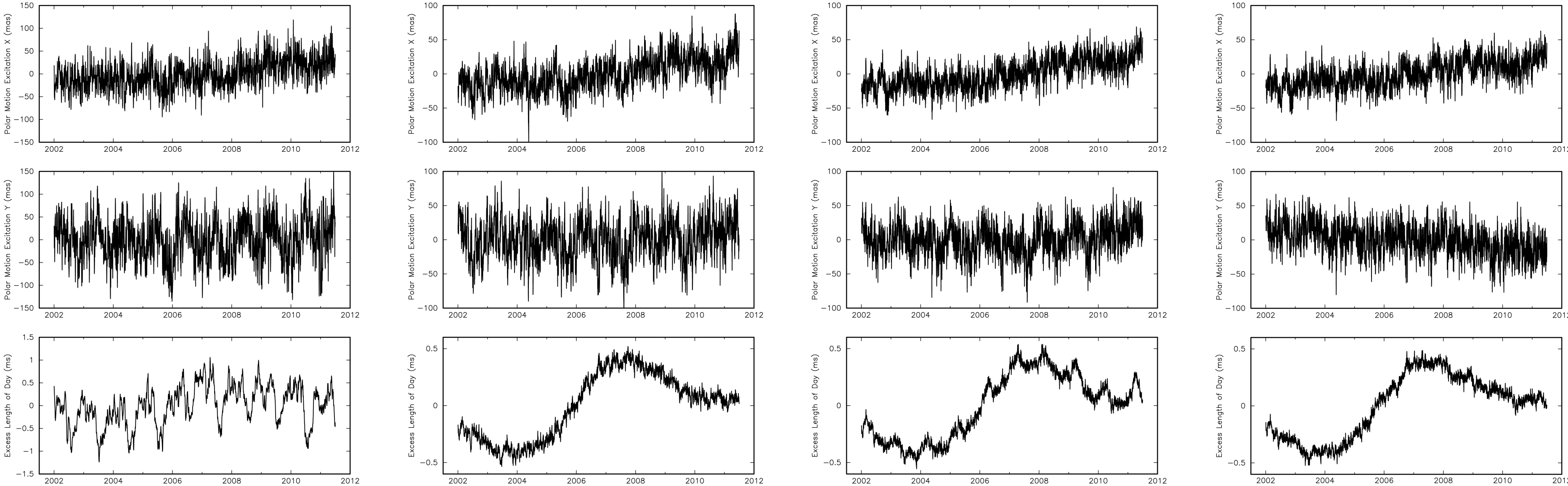
Modeled Change in Earth Rotation

	Δlod (μsec)	$\Delta\chi_x$ (mas)	$\Delta\chi_y$ (mas)
Recent great earthquakes			
2004 Sumatra (M_w = 9.3)	–6.77	–1.41	1.85
2005 North Sumatra (M_w = 8.5)	–0.84	–0.25	0.08
2007 South Sumatra (M_w = 8.4)	–0.64	–0.17	–0.07
2010 Chile (M_w = 8.8)	–1.71	–1.38	3.26
2011 Japan (M_w = 9.0)	–1.36	–2.97	3.27
Approximate measurement uncertainty (1σ)	10	5	5
Other great earthquakes			
1960 Chile (M_w = 9.5; Chao & Gross, 1987)	–8.40	–9.53	20.45
1964 Alaska (M_w = 9.2; Chao & Gross, 1987)	6.79	–7.11	–2.31

Observed Residual Excitation

(2002.0 – 20011.5)

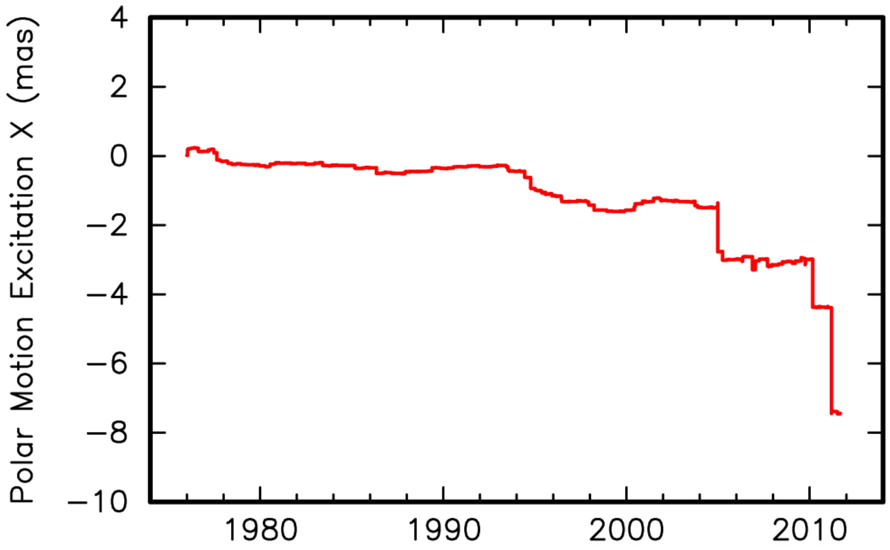
observed	obs-atm	obs-atm-ocn	obs-atm-ocn-hyd

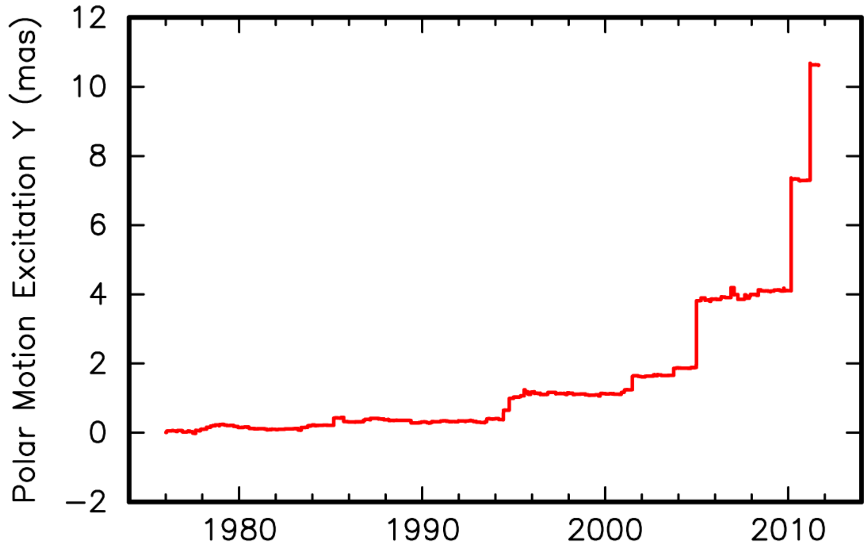


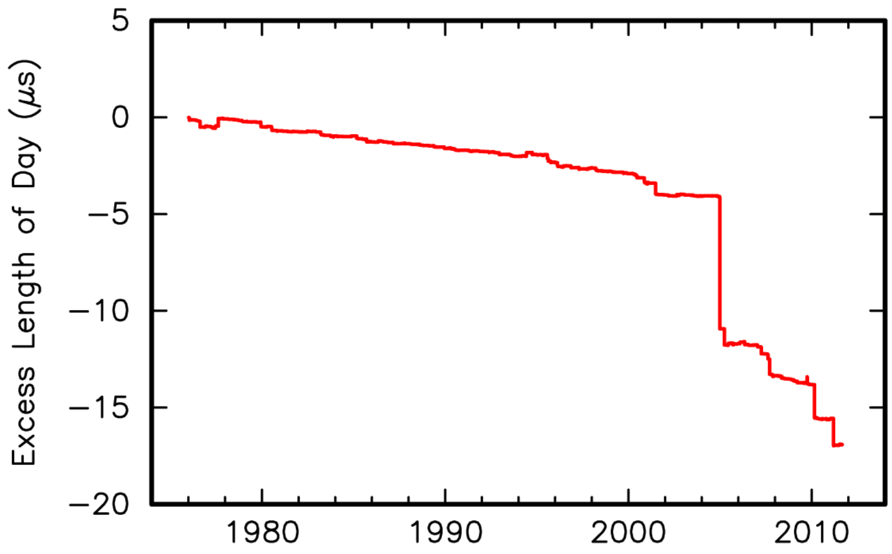
Variance (percent reduction)

	observed	obs-atm	obs-atm-ocn	obs-atm-ocn-hyd
chi-x:	912 mas^2	564 mas^2 (38%)	464 mas^2 (49%)	394 mas^2 (57%)
chi-y:	1978 mas^2	841 mas^2 (57%)	532 mas^2 (73%)	503 mas^2 (75%)
lod:	0.185 ms^2	0.0798 ms^2 (57%)	0.0807 ms^2 (56%)	0.0745 ms^2 (60%)

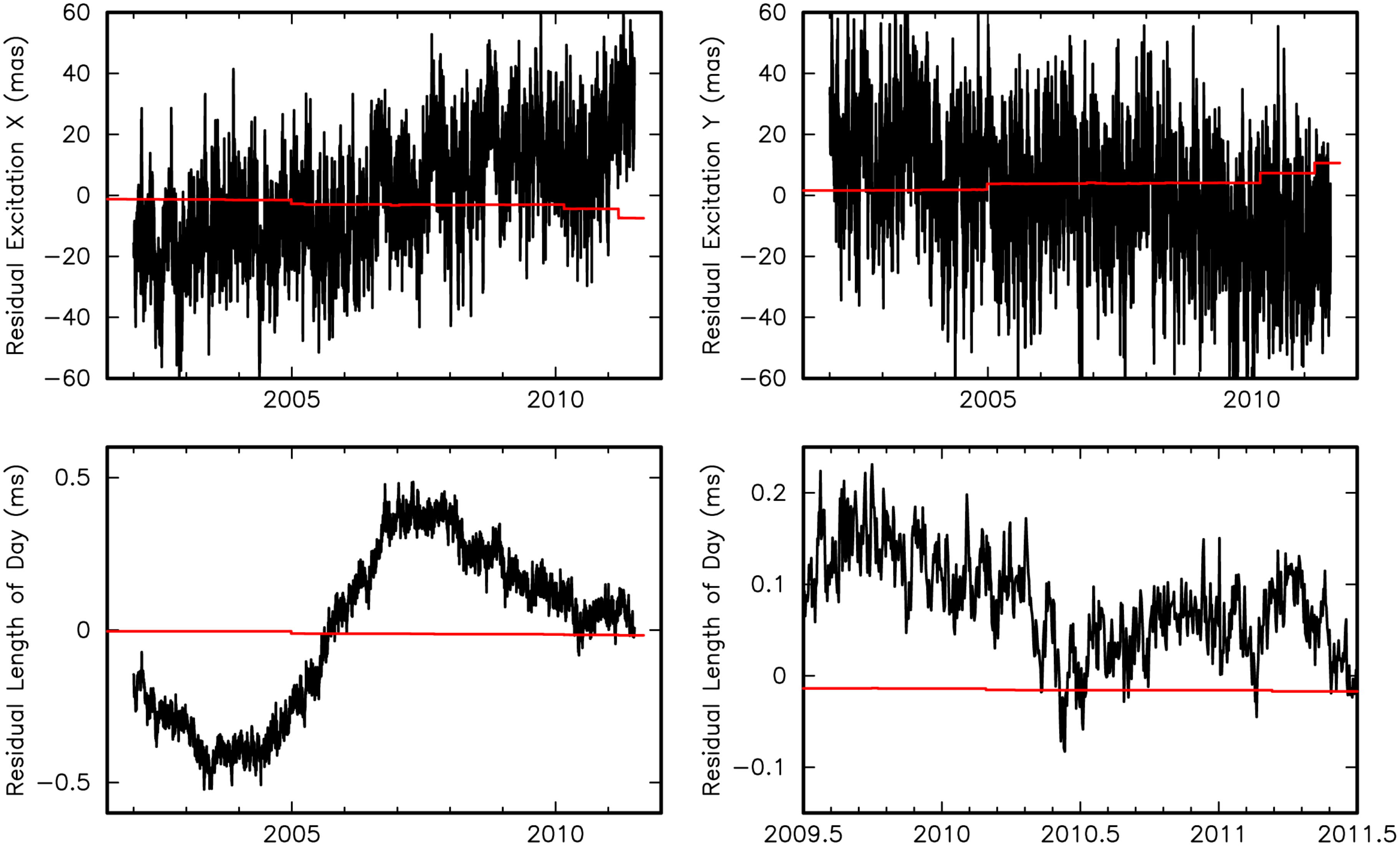
Modeled Change in Earth Rotation





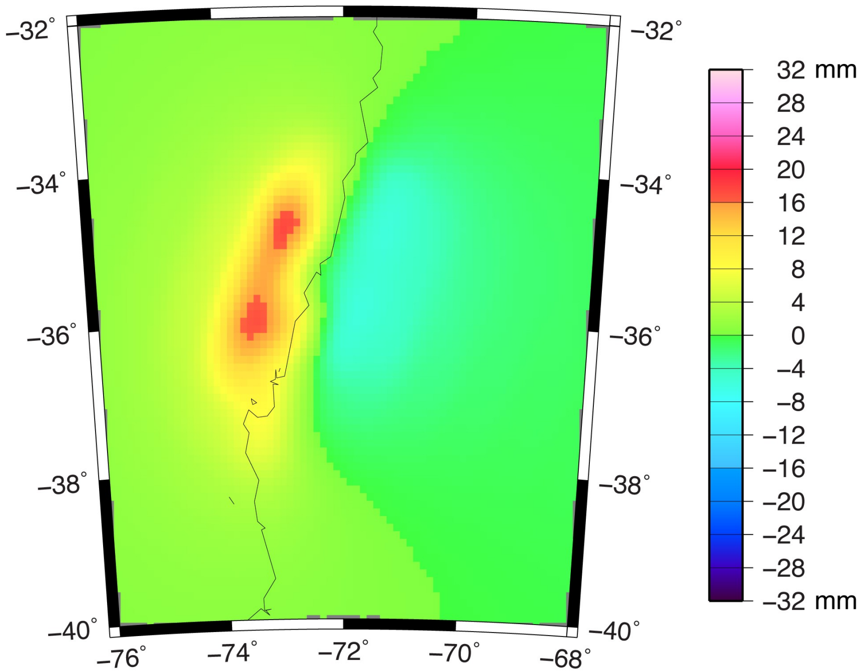


Observed and Modeled Change in Rotation

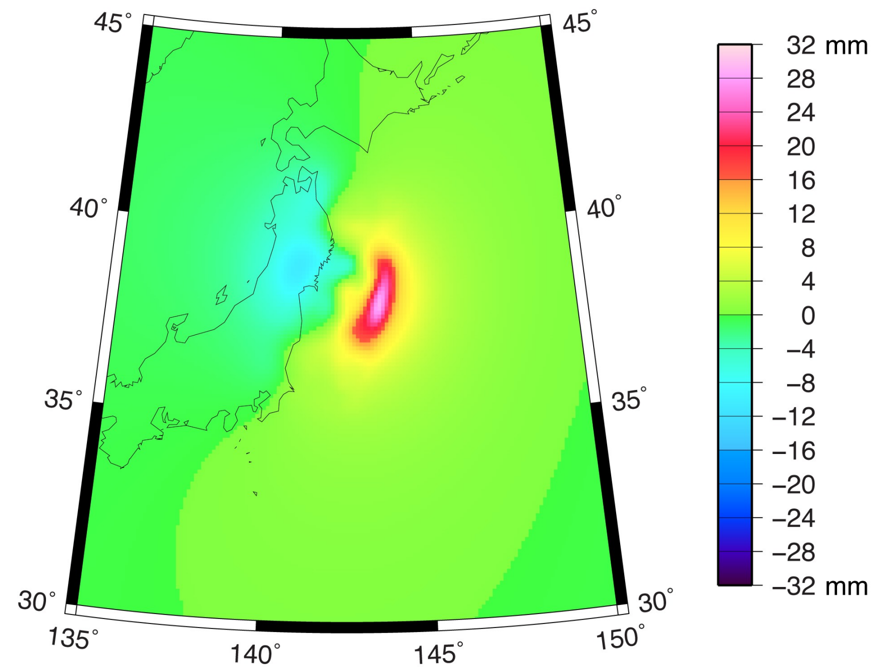


Modeled Change in Geoid

2010 Chile



2011 Japan



Modeled Change in Geopotential

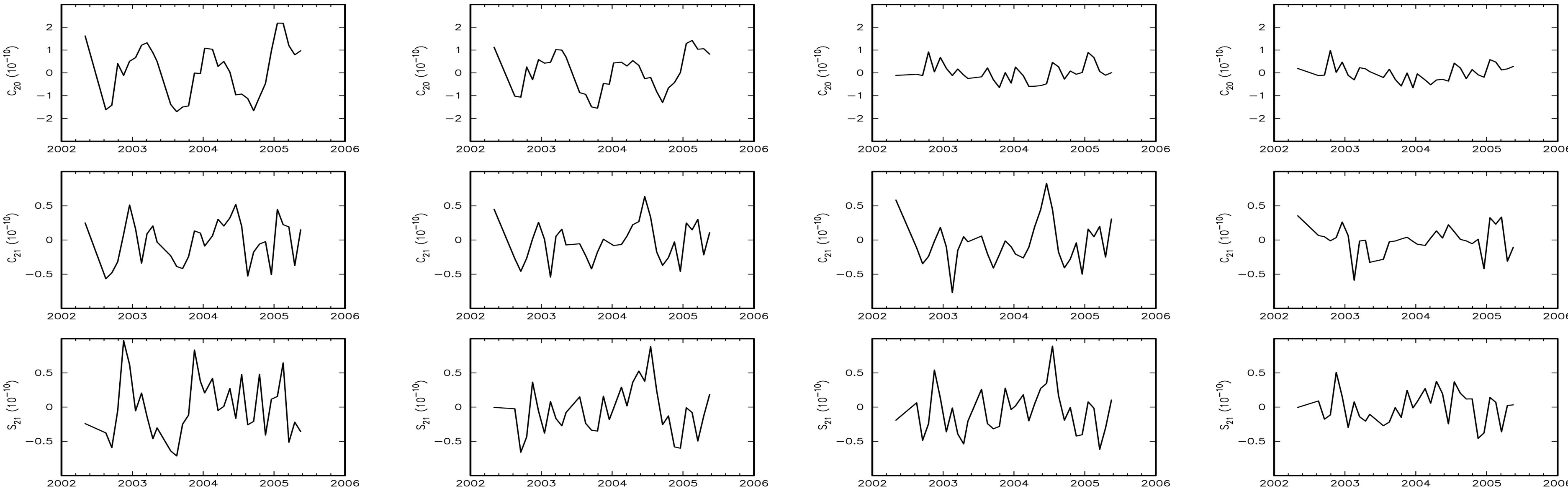
		ΔJ_2 (10^{-11})	ΔJ_3 (10^{-11})	ΔJ_4 (10^{-11})	ΔJ_5 (10^{-11})
2004 Sumatran earthquake					
2004 Sumatra	(M_w = 9.3)	–2.368	–0.621	0.254	0.305
2005 North Sumatra	(M_w = 8.5)	–0.283	–0.043	0.034	0.033
2007 South Sumatra	(M_w = 8.4)	–0.216	0.000	0.031	0.024
2010 Chile	(M_w = 8.8)	–0.302	0.608	–0.263	–0.018
2011 Japan	(M_w = 9.0)	0.006	–0.560	–0.343	–0.116
Approximate SLR measurement uncertainty (1σ)		1.3	1.6	4.9	
Other great earthquakes					
1960 Chilean	(M_w = 9.5; Chao & Gross, 1987)	–0.83	3.29	–1.89	3.64
1964 Alaskan	(M_w = 9.2; Chao & Gross, 1987)	5.25	2.35	1.40	1.62

$$\Delta J_l = -\sqrt{2l+1}\,\Delta C_{l0}$$

Observed Residual Potential Coefficients from Satellite Laser Ranging

(2002.3 – 2005.4)

observed | obs–(atm+ocn) | obs–(all surface) | nonseasonal obs–(all surface)

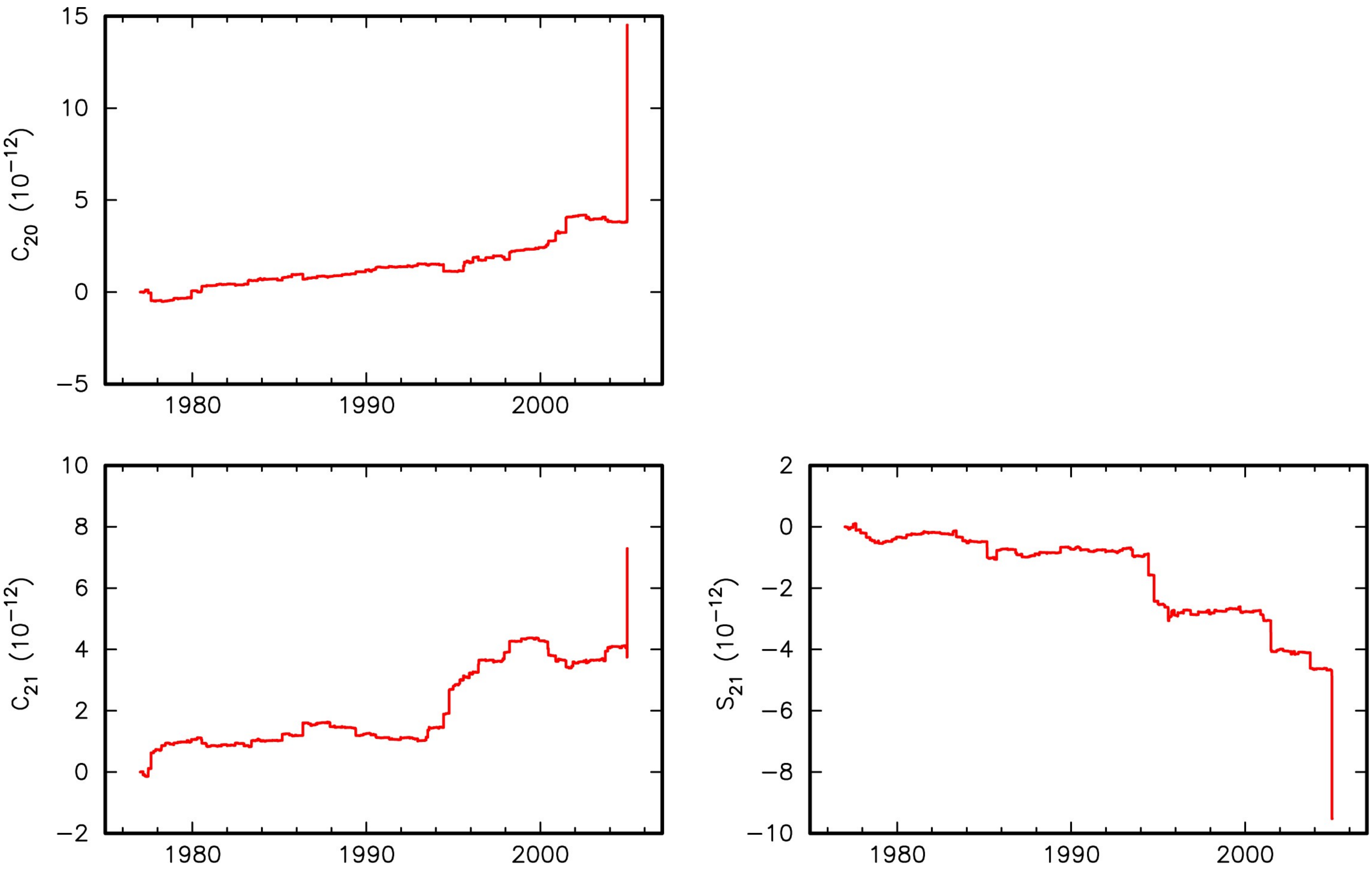


Variance (percent reduction)

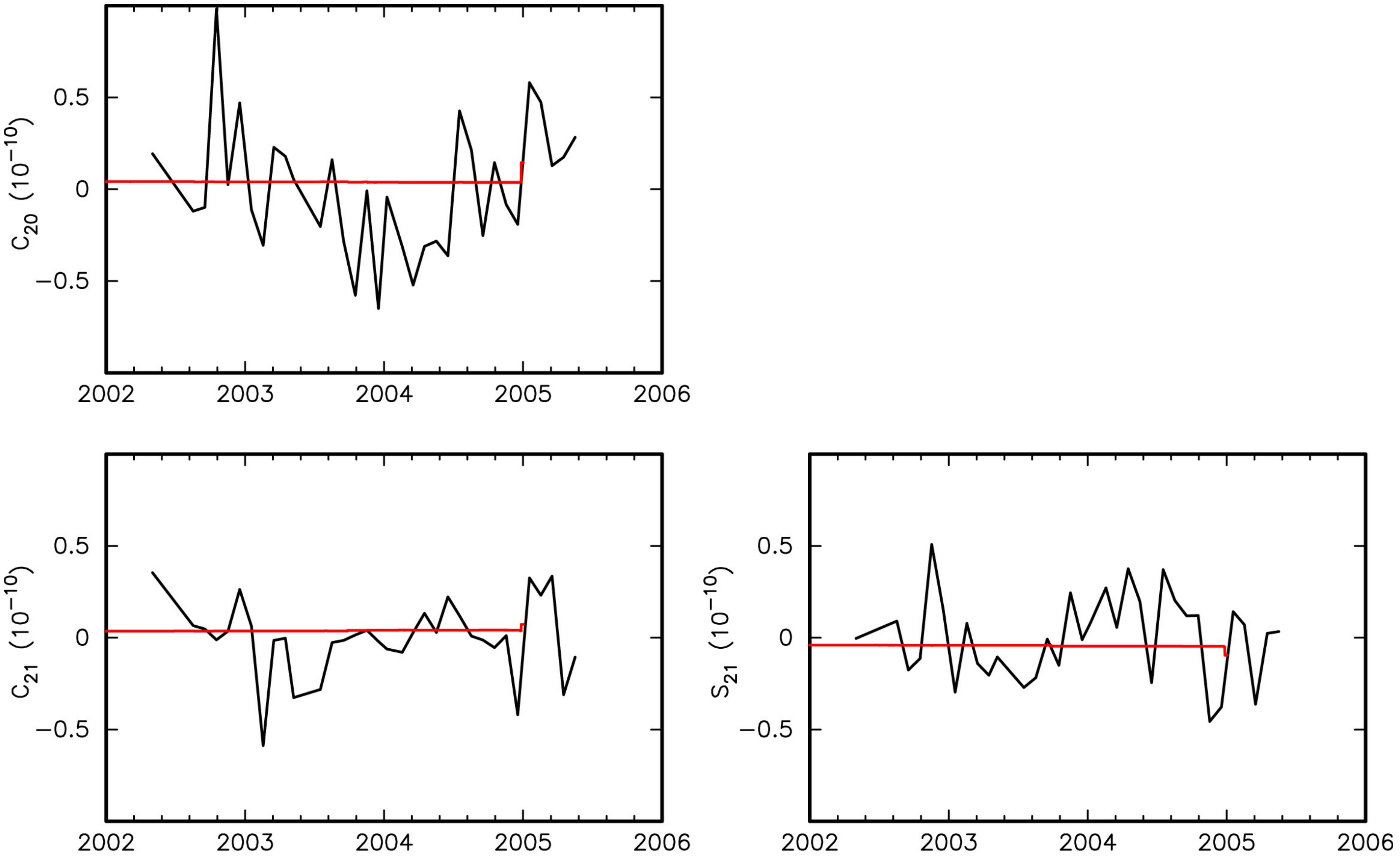
	observed	obs–(atm+ocn)	obs–(all surface)	nonseasonal obs–(all surface)
C_{20}:	134.55	72.94 (46%)	15.74 (88%)	12.32 (91%)
C_{21}:	9.73	7.64 (21%)	10.30 (–6%)	4.22 (57%)
S_{21}:	18.30	11.70 (36%)	10.55 (42%)	5.28 (71%)

units of variance: 10^{-22}

Modeled Change in Geopotential



Observed and Modeled Change in Potential



Summary

- Changes in Earth's shape, rotation, and gravity
  - Are measured by the same observing system
    - SLR, GPS, DORIS, VLBI (shape and rotation)
  - Often have a common cause
    - Atmosphere, oceans, hydrology, earthquakes, global isostatic adjustment
  - But are sometimes modeled separately
    - Flat Earth models for earthquake displacements, spherical for rotation
- Unified model
  - Allows common shape, rotation, and gravity observations to be used to determine model parameters
  - Allows consistent geodetic modeling of
    - Surface change
    - Mass transport and exchange
    - Angular momentum exchange

Summary, cont.

- Earthquakes redistribute the Earth’s mass on a global scale
  - Change the Earth’s rotation and gravitational field
- Greatest earthquakes have greatest effect
  - 1960 Chile
    - 23 mas change in polar motion excitation
    - 8 μs change in length of day
- Current observing systems are accurate enough to detect changes caused by next great earthquake
  - Polar motion excitation accuracy about 5 mas
  - LOD accuracy about 10 μs