Recent advances in applications of geodetic VLBI to geophysics

Véronique Dehant, Sébastien Lambert, Laurence Koot, Antony Trinh, Marta Folgueira

Royal Observatory of Belgium, Paris Observatory, Complutense University of Madrid

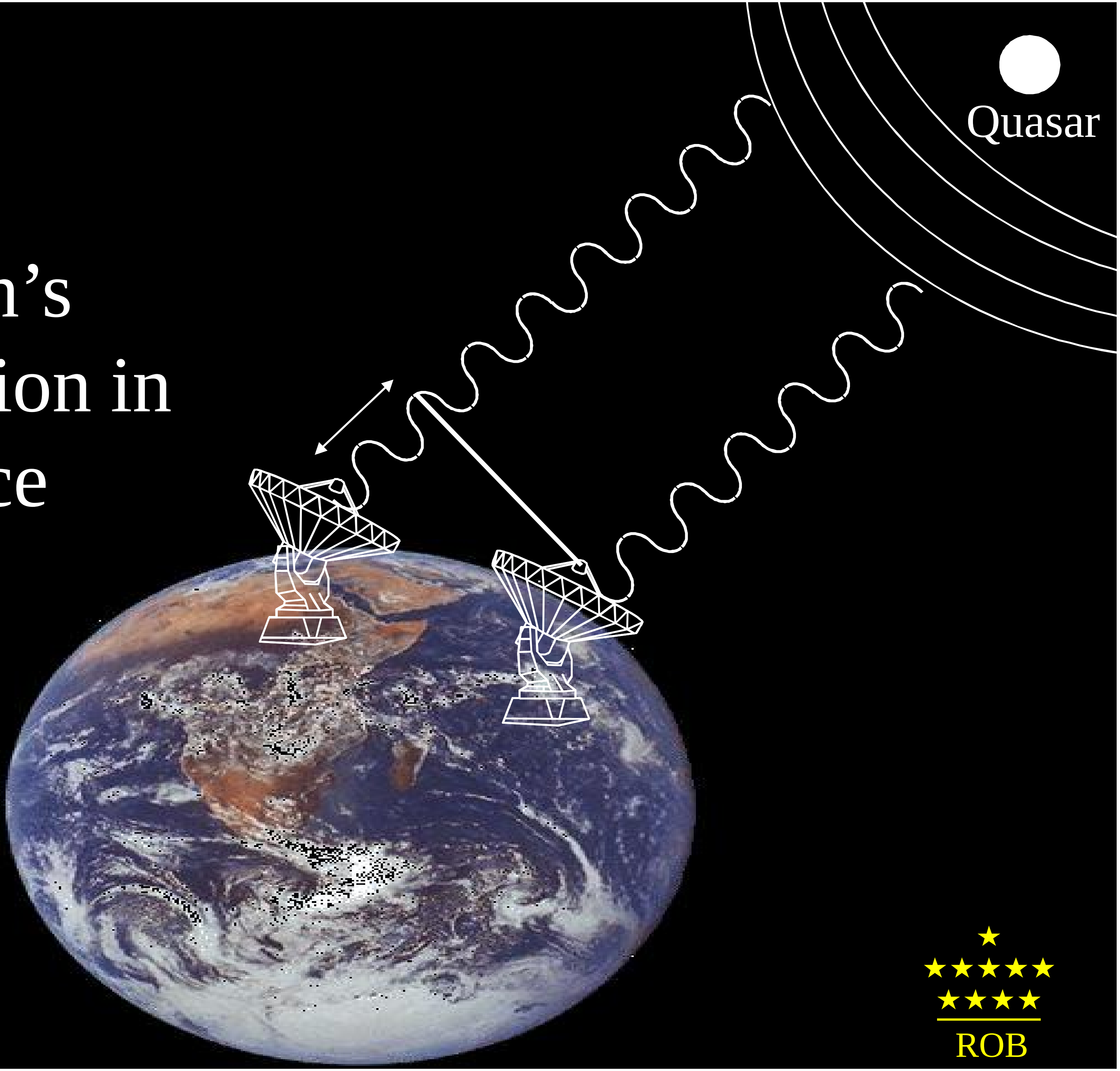
ROB

Nutation measured by VLBI

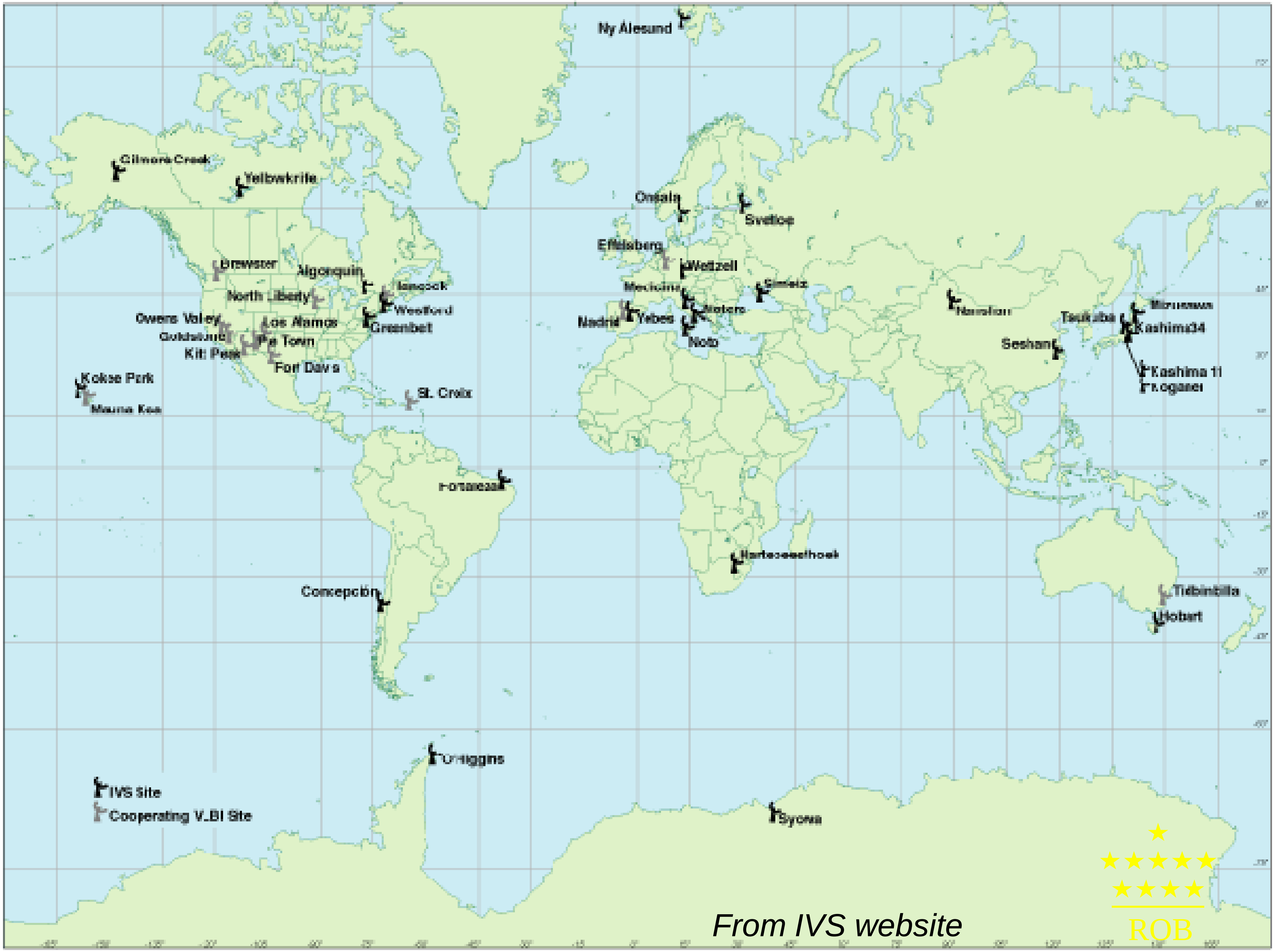
Noto station
From http://www.noto.ira.inaf.it/

ROB

Earth's orientation in space



ROB

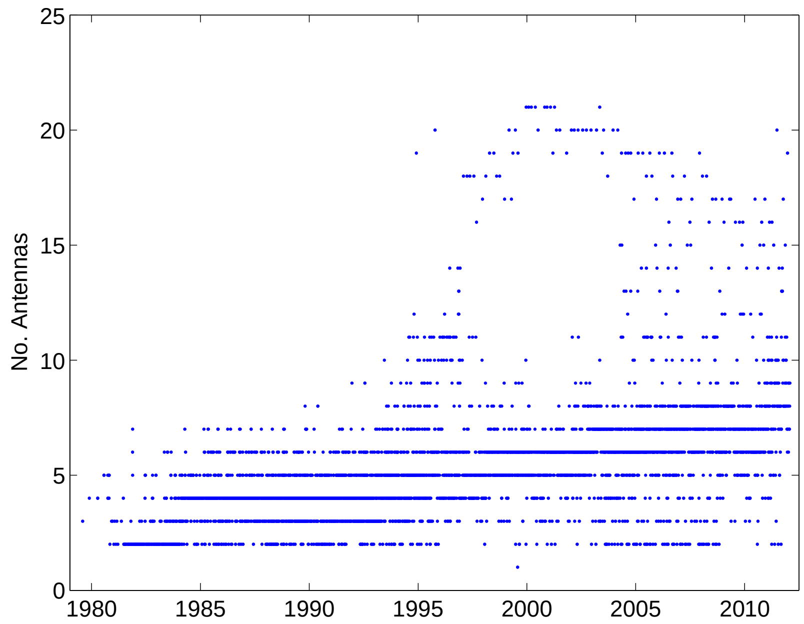


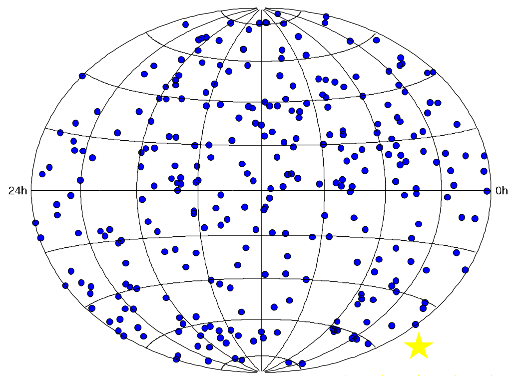
From IVS website


ROB

Recent Advances in observation

- Network
  - More stations
  - More extended networks
  - More sources observed in each session
  - Upcoming VLBI 2010
- Reference frames
  - ITRF 20xx
  - ICRF2 (Ma et al. 2009)
    - Stronger set of defining sources
    - Better coverage of both hemispheres
    - Improved stability of the axes (10 μas)



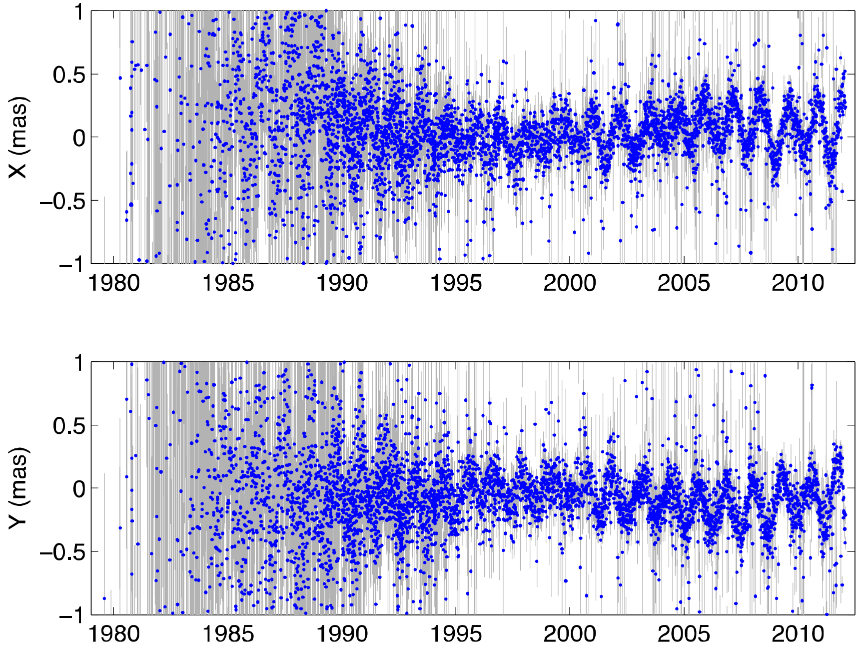


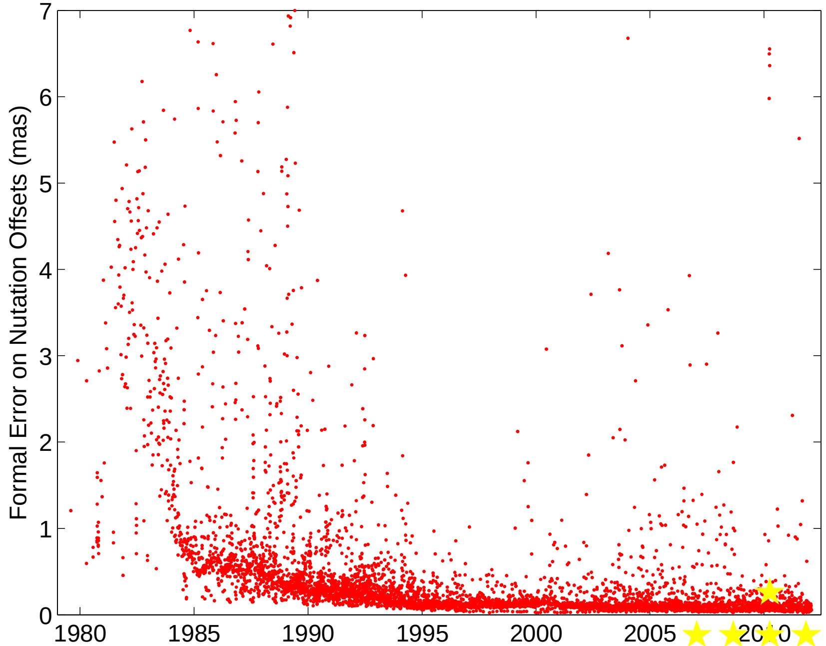
ICRF2 defining sources

ROB

Nutation Series

- Longer time series
  - Better adjustment of long-period terms (e.g., 18.6-yr)
  - Improvement of the formal error
  - Can choose to drop data before 1995



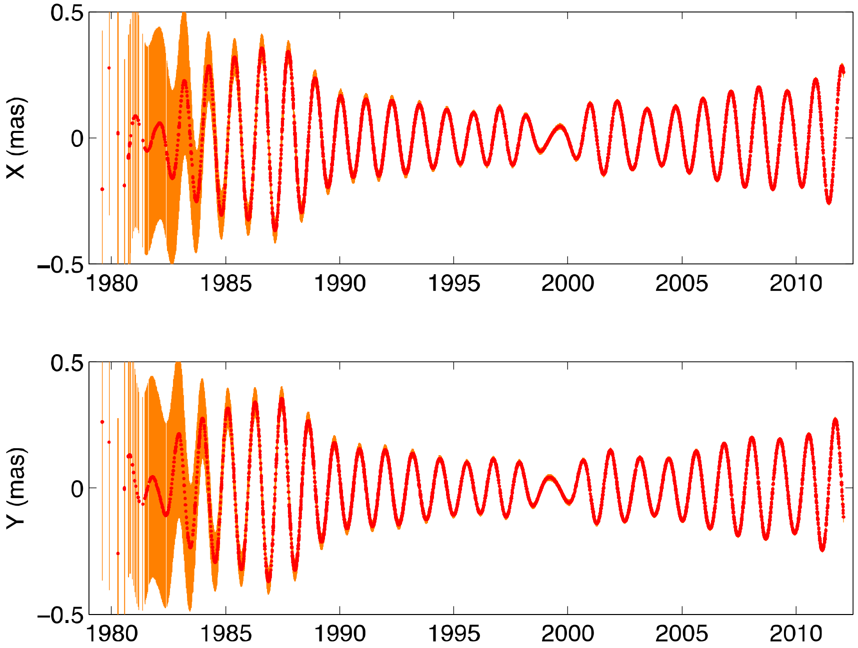


ROB

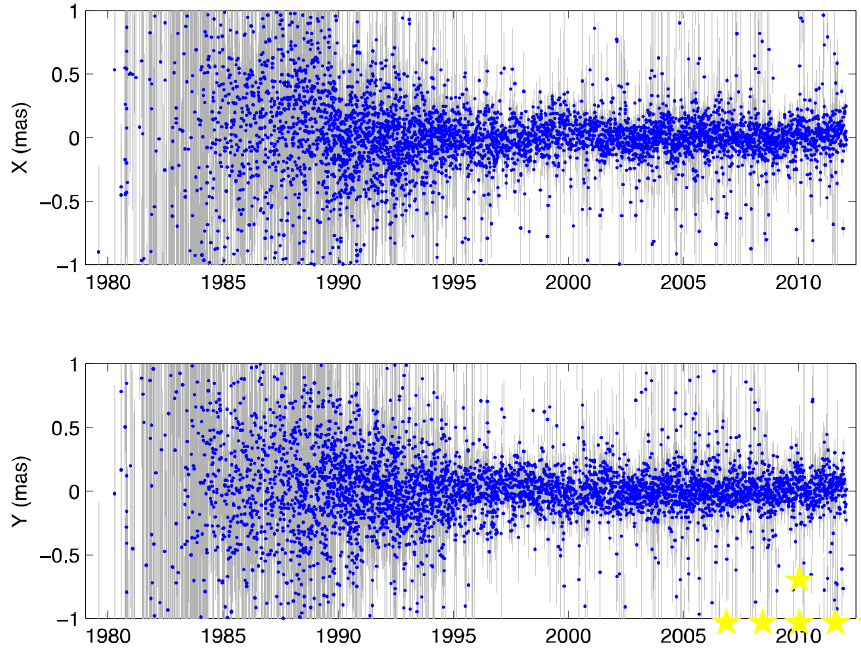
Some Challenges for the free nutations

- The amplitudes can only be observed due to the poor knowledge of their excitation! Only FCN free mode observed. (but resonance)
- Explain the variations of the FCN amplitude/phase
- Detect a signal related to the FICN (see poster of Lambert et al.)

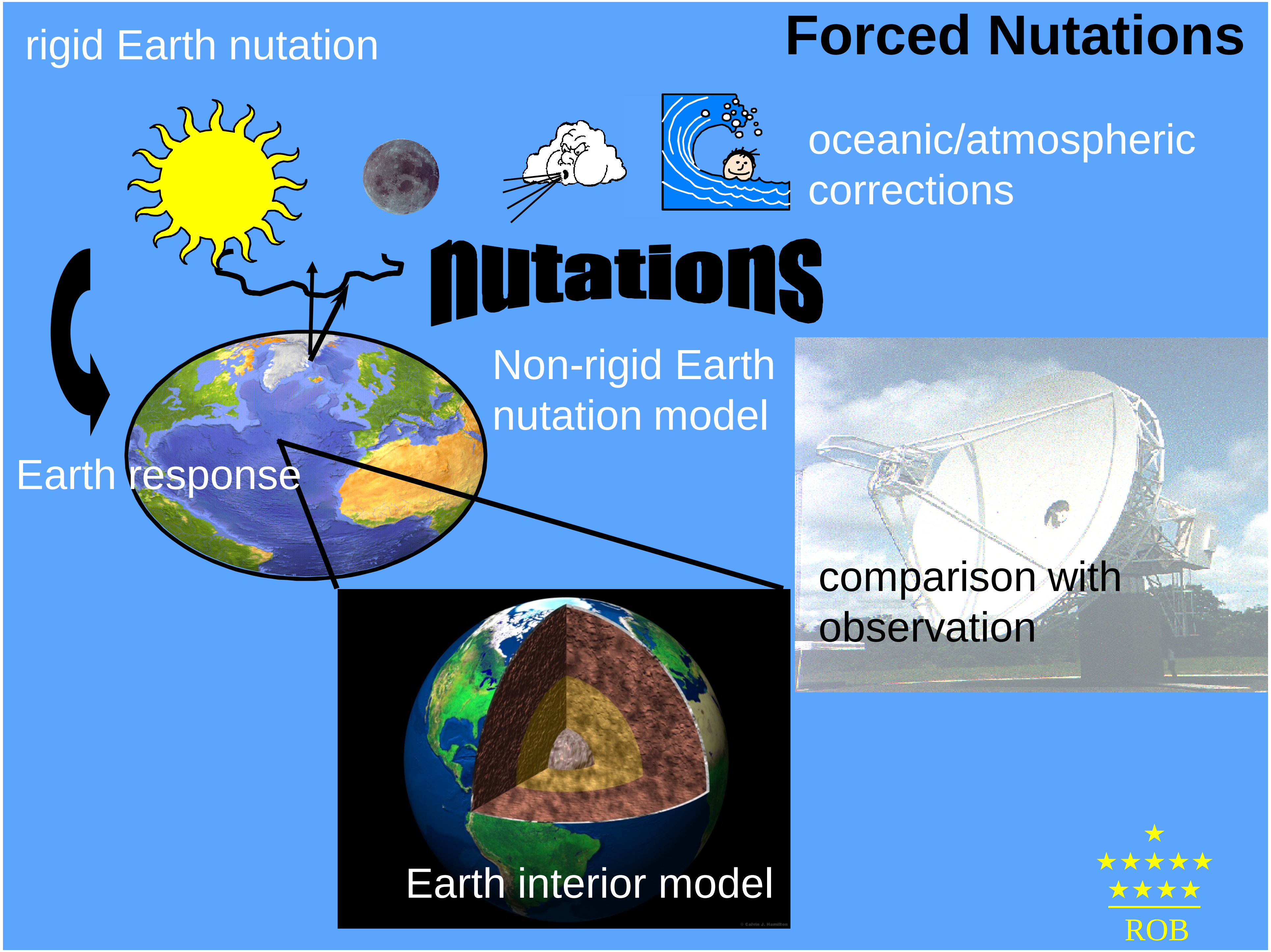
The FCN



The nutation after removal of the FCN and main tidal terms

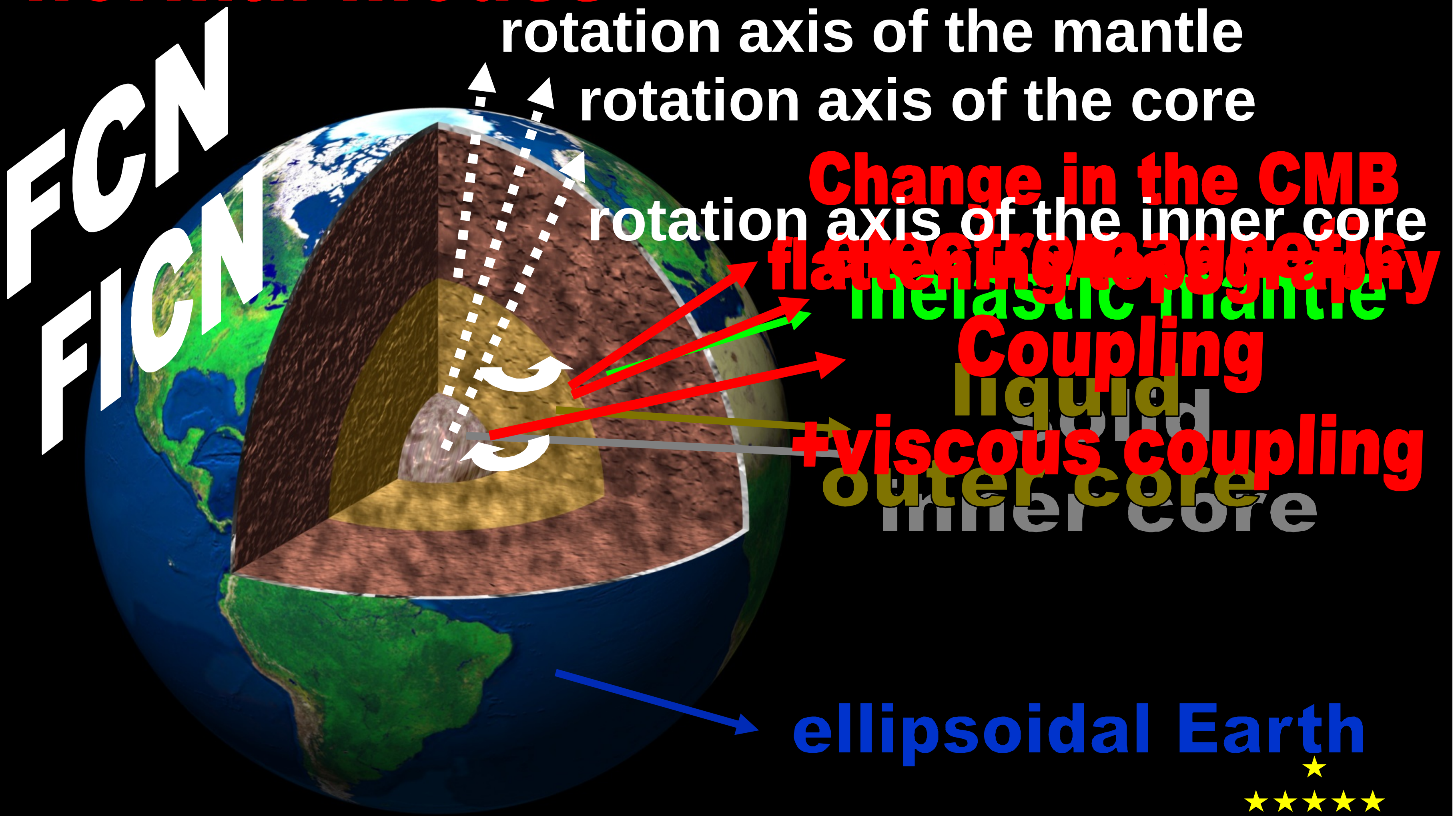


ROB


Forced Nutations
rigid Earth nutation
oceanic/atmospheric corrections
nutations
Non-rigid Earth nutation model
Earth response
comparison with observation
Earth interior model
ROB

structure of the Earth's interior for its response

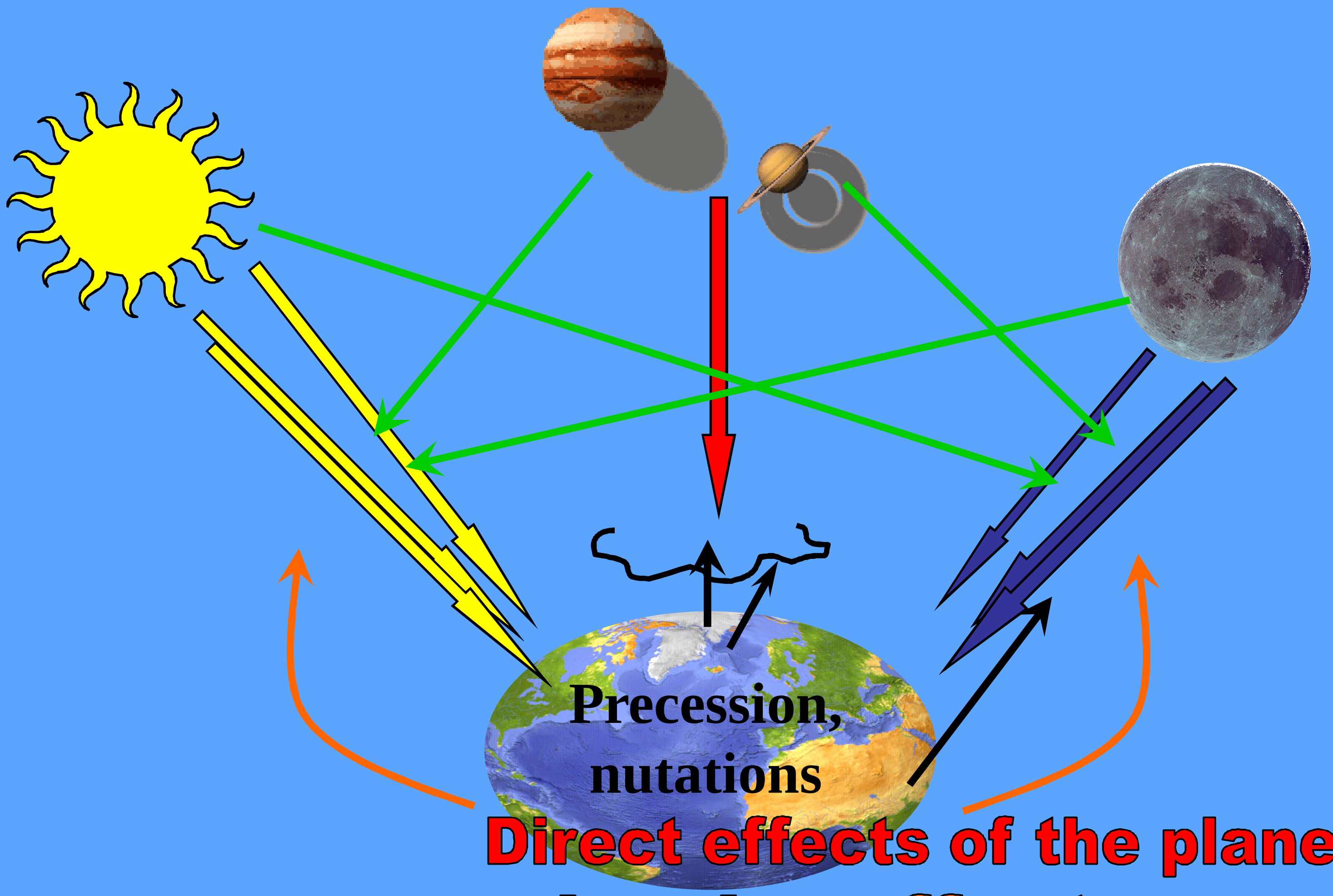
+ normal modes



ROB


ROB

Rigid Earth nutation theory



Direct effects of the planets

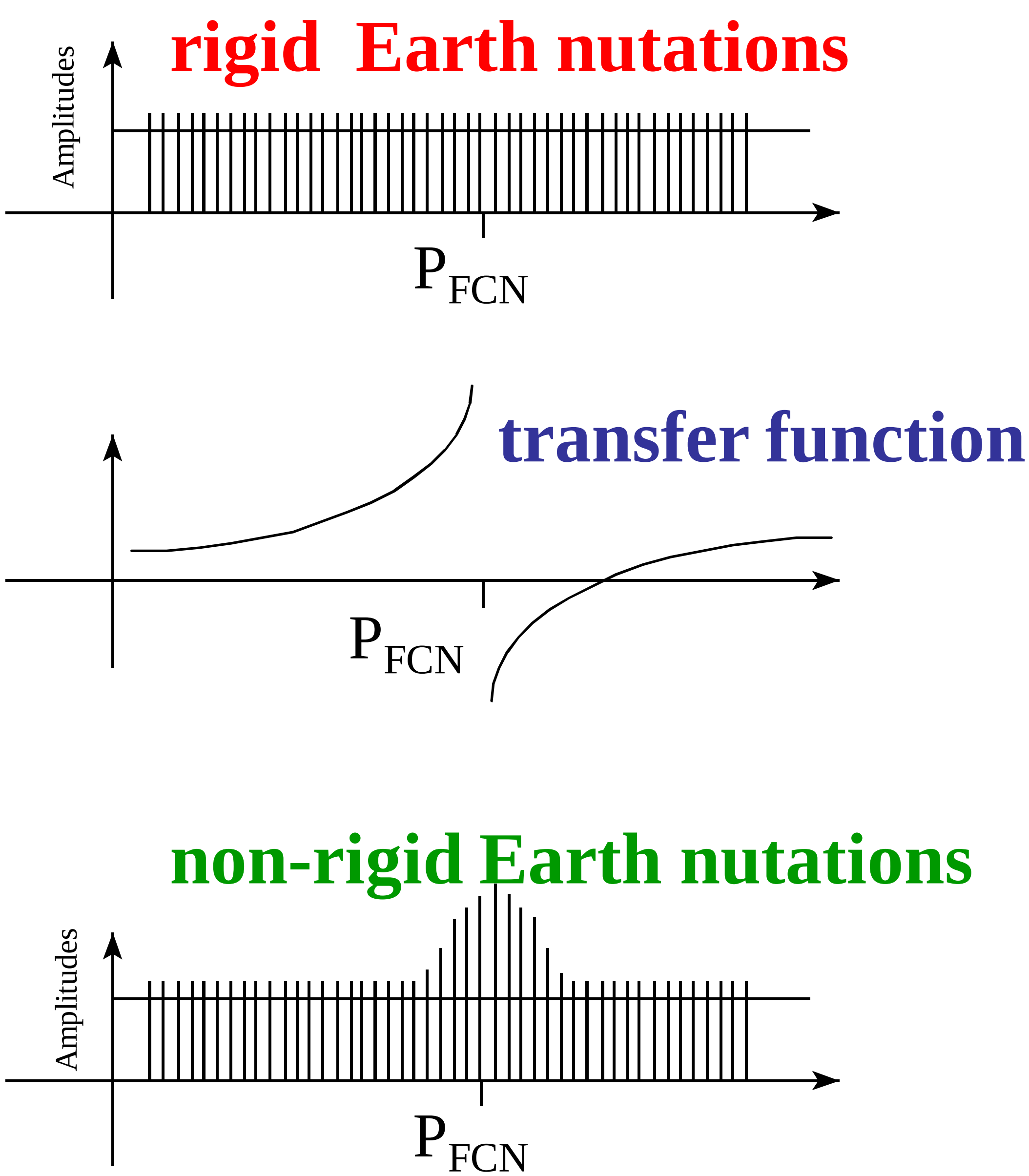
Indirect effects of the planets
on the relative positions of the Earth, Moon and Sun

second order effects

Relative position of the Earth with respect to the Sun

Relative position of the Moon with respect of the Earth

ROB



1. calculate rigid nutations (precision better than observation precision) from **celestial mechanics**
2. Calculate response of planet (transfer function in frequency domain) from **geophysics**

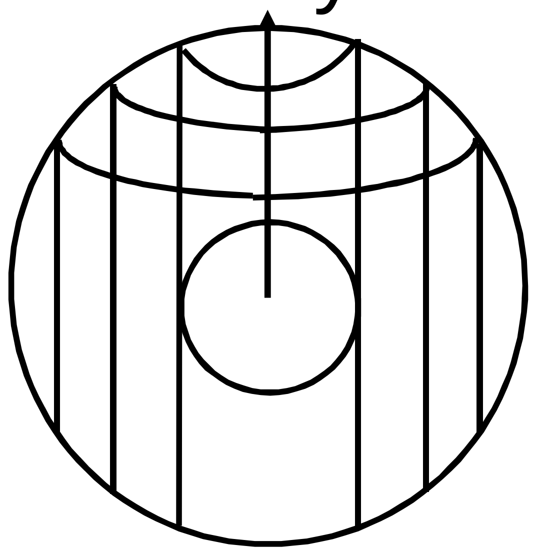
↓

Earth: **amplifications** up to 30 mas

ROB

Observed magnetic field and its secular variations ➔flow at CMB ➔ velocity field in the core

☞ flow as rigid rotation of coaxial cylinders along Earth rotation axis (Taylor cylinders) :



☞ torsional wave linking all the cylinders (quasi-Taylor state)

Braginsky 1970, Jault et al. 1988

ROB

Earth rotation changes due to the core; core-mantle coupling

→ coupling mechanisms:

- ☞ topographic torque
- ☞ gravitational torque
- ☞ viscous torque
- ☞ electromagnetic torque

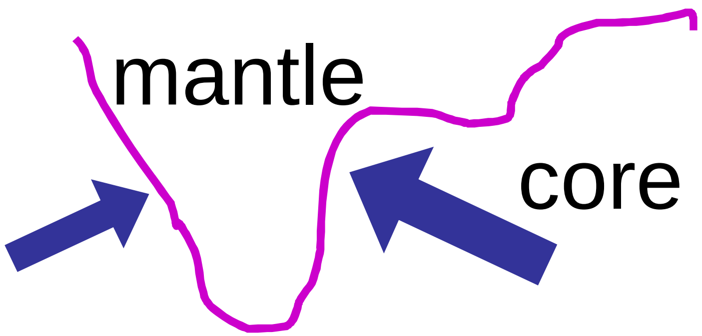
Core Angular Momentum exchange due to **topographic** torque at CMB

☞ pressure at CMB

☞ core-mantle boundary topography (<2km)

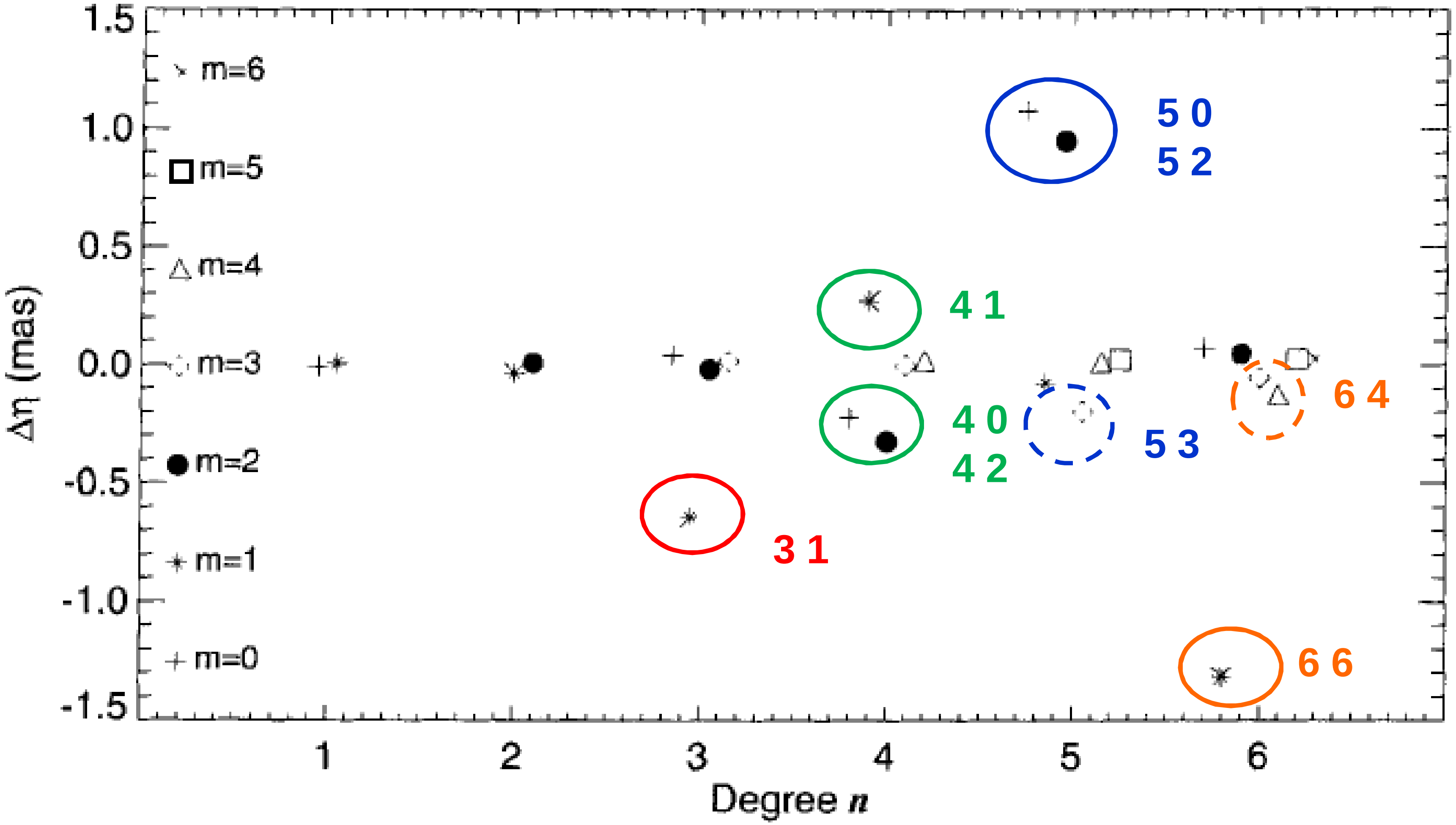
Difficult, challenging, controversial

but cannot be ruled out



e.g. Hide 1977

ROB



From Wu and Wahr, 1997

ROB

Topographic coupling

- Why only some of the topography coefficients are important?
- Related to resonance with inertial waves

  [*when perturbing a rotating fluid, the particle motion is characterized by a low-frequency oscillation called inertial wave*]
- Related to the geometry of the core and of the topography

  ☞ **Analytical approach** *work with Mihaela Puica*

  ☞ **Numerical approach** *work with Quentin Geerinckx*

Research objective and strategy

- Aim at obtaining torque and associated effects on nutation
- Strategy:
  1
  - Establish the motion equations and boundary conditions in the fluid;
  - Compute analytically/numerically the solutions;
  - Obtain the dynamic pressure as a function of the physical parameters;
  - Determine the topographic torque.
- Assessment: Comparison with Wu and Wahr (1997) who used a numerical technique

ROB

Differential equations and boundary conditions

- Linearized Navier-Stokes equation:

The oscillations of a rotating fluid where the restoring force involves Coriolis force are inertial waves (frequency lower than 2ω)

$$\begin{cases} i\sigma_m \vec{q} + 2\vec{\hat{z}} \times \vec{q} + \nabla\Phi = 0 \\ \nabla \cdot \vec{q} = 0 \\ \vec{n} \cdot \vec{q} + \Omega^{-1} L^{-1} \vec{n} \cdot \vec{v} = 0 \end{cases}$$

Coriolis

where $\Phi = \frac{\phi}{\Omega^2 L^2}$ and $\phi = \frac{p}{\rho_f} + \chi$.

ROB

Process for obtaining the solutions and the torque

- $\vec{q}$ as a function of $\nabla\Phi$: $\vec{q} = \dfrac{-i\,\sigma_m}{4-\sigma_m^2}\left[\nabla\Phi - \dfrac{2}{i\,\sigma_m}\,\vec{\hat{z}}\times\nabla\Phi - \dfrac{4}{\sigma_m^2}\,(\vec{\hat{z}}\cdot\nabla\Phi)\,\vec{\hat{z}}\right]$

- Expression for Φ: $\Phi = \sum\limits_{l=1} a_l^k\, P_{lk}\!\left(\dfrac{\sigma_m}{2}\right) Y_l^k(\vartheta,\lambda).$

- Expression of $\vec{v}$ in function of χ:

$$
\begin{aligned}
v_1 + i v_2 &= -\frac{i}{\Omega}\left(\frac{\partial\chi}{\partial x} + i\frac{\partial\chi}{\partial y}\right)\\
v_3 &= \frac{i}{\Omega}\frac{d\chi}{dz}
\end{aligned}
$$

$$
\vec{\Gamma}_{topo} = \vec{\Gamma}_0 + \int\!\!\int_{CMB} \vec{r}\times\vec{n}\rho_f(\chi+\phi)\,dS = \vec{\Gamma}_0 + \vec{\Gamma}^{\chi}_{topo} + \vec{\Gamma}^{\phi}_{topo}
$$

$$
\begin{aligned}
\Gamma_1^\phi &= -\frac{i}{2}\sum_{l=1}^{l_{max}}\sum_{k=-l}^{l}(-1)^k\left[\sqrt{l-k}\sqrt{l+k+1}\,\epsilon_l^{-k-1} + \sqrt{l-k+1}\sqrt{l+k}\,\epsilon_l^{-k+1}\right] P_{lk}\!\left(\frac{\sigma_m}{2}\right) a_l^k\\
\Gamma_2^\phi &= \frac{1}{2}\sum_{l=1}^{l_{max}}\sum_{k=-l}^{l}(-1)^k\left[-\sqrt{l+k+1}\sqrt{l-k}\,\epsilon_l^{-k-1} + \sqrt{l-k+1}\sqrt{l+k}\,\epsilon_l^{-k+1}\right] P_{lk}\!\left(\frac{\sigma_m}{2}\right) a_l^k
\end{aligned}
$$

ROB

Final expressions

- Equation for obtaining the analytical expressions of a_l^k in function of the topography at the CMB ε_n^m

$$
\begin{aligned}
&\sin^2\vartheta \sum_{l,k} Y_l^k \left[k P_{lk}\left(\frac{\sigma_m}{2}\right) - \left(1-\frac{\sigma_m^2}{4}\right) P'_{lk}\left(\frac{\sigma_m}{2}\right)\right] a_l^k \\
&+ \sin^2\vartheta \left[2\sqrt{\frac{2\pi}{15}}\left(1+3\sum_{n=0}\epsilon_n^m Y_n^m\right)\left(\frac{(\sigma_m^2+\sigma_m-2)}{2\sigma_m} Y_2^1 m_f^- + \frac{(-\sigma_m^2+\sigma_m+2)}{2\sigma_m} Y_2^{-1} m_f^+\right)\right. \\
&\left. + \sqrt{\frac{2\pi}{3}}\frac{(4-\sigma_m^2)}{2\sigma_m}\Psi\left(-Y_1^1 m_f^- + Y_1^{-1} m_f^+\right)\right] + \cos^2\vartheta\sqrt{\frac{2\pi}{3}}\,\Psi\left(-\frac{(\sigma_m+2)}{2}Y_1^1 m_f^- + \frac{(\sigma_m-2)}{2}Y_1^{-1}m_f^+\right) \\
&+ \sqrt{\frac{2\pi}{15}}\sum_{n=1} m\epsilon_n^m Y_n^m \left(\frac{(\sigma_m+2)}{2}Y_2^1 m_f^- + \frac{(\sigma_m-2)}{2} Y_2^{-1} m_f^+\right) = 0
\end{aligned}
$$

$Y_l^k \equiv Y_l^k(\vartheta,\lambda)$, $m_f^+ = m_1^f + i m_2^f$, $m_f^- = m_1^f - i m_2^f$, $P'_{lk}(x) = \frac{dP_{lk}(x)}{dx}$ and

$$
\Psi = \sum_{n=1}\epsilon_n^m\left[\frac{n\sqrt{n-m+1}\sqrt{n+m+1}}{\sqrt{2n+1}\sqrt{2n+3}}Y_{n+1}^m - \frac{(n+1)\sqrt{n-m}\sqrt{n+m}}{\sqrt{2n+1}\sqrt{2n-1}}Y_{n-1}^m\right]
$$

Solutions

- Expressions for a_l^k involving $\varepsilon_n^m Y_n^m$ Y_2^0 Y_2^1
- No radius dependence except for a global scaling
- This yield coupling such as

$$Y_{n\pm 4}^{m\pm 1}$$

- This thus involves particular harmonics
- Also the solutions contain polynomial in the frequency σ.

ROB

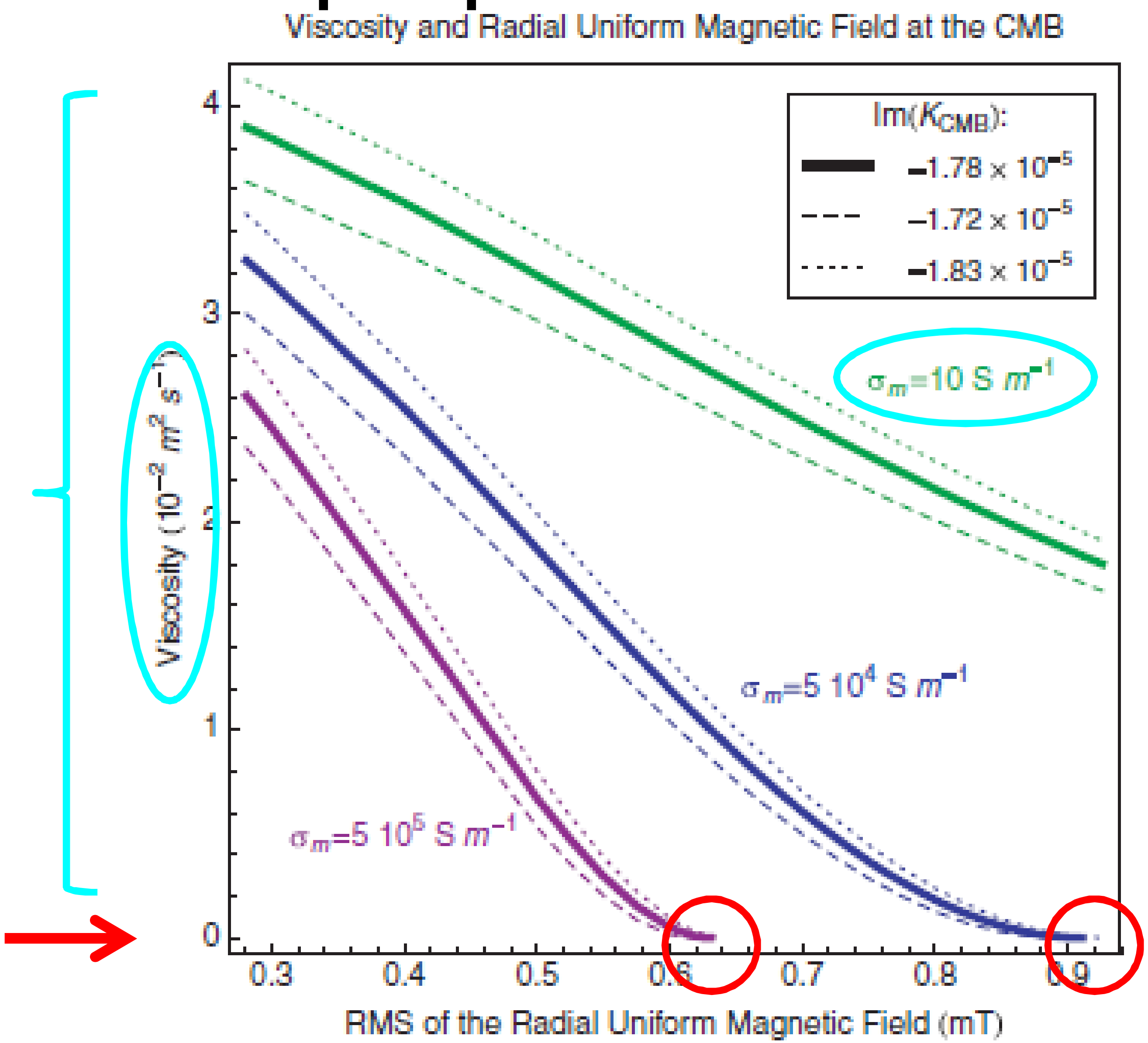
Electromagnetic torque + viscous torque: dissipative

- Outer core electrical conductivity: known from laboratory experiments: **$5\ 10^5$ S m^{-1}** (Stacey & Anderson 2001).
- Lowermost mantle electrical conductivity (~200 m layer at the base of the mantle): unknown but has to be lower than that of the core.

  σ_m= 10 S m^{-1}, $5\ 10^4$ S m^{-1}, $5\ 10^5$ S m^{-1}
- RMS of the radial magnetic field at the CMB: from surface magnetic field measurements: > 0.3 mT.
- Viscosity of the outer core fluid close to the CMB:
  - molecular viscosity: ~10^{-6} m^2 s^{-1} (laboratory experiments and ab initio computations).
  - eddy viscosity: < 10^{-4} m^2 s^{-1} (Buffett & Christensen 2007).

ROB

Constraints on the physical properties of the CMB



Coupling model used: Buffet et al. 2002 for EM and Mathews & Guo 2005 for viscomagnetic

From Koot et al. 2010

ROB

Constraints on the physical properties of the CMB

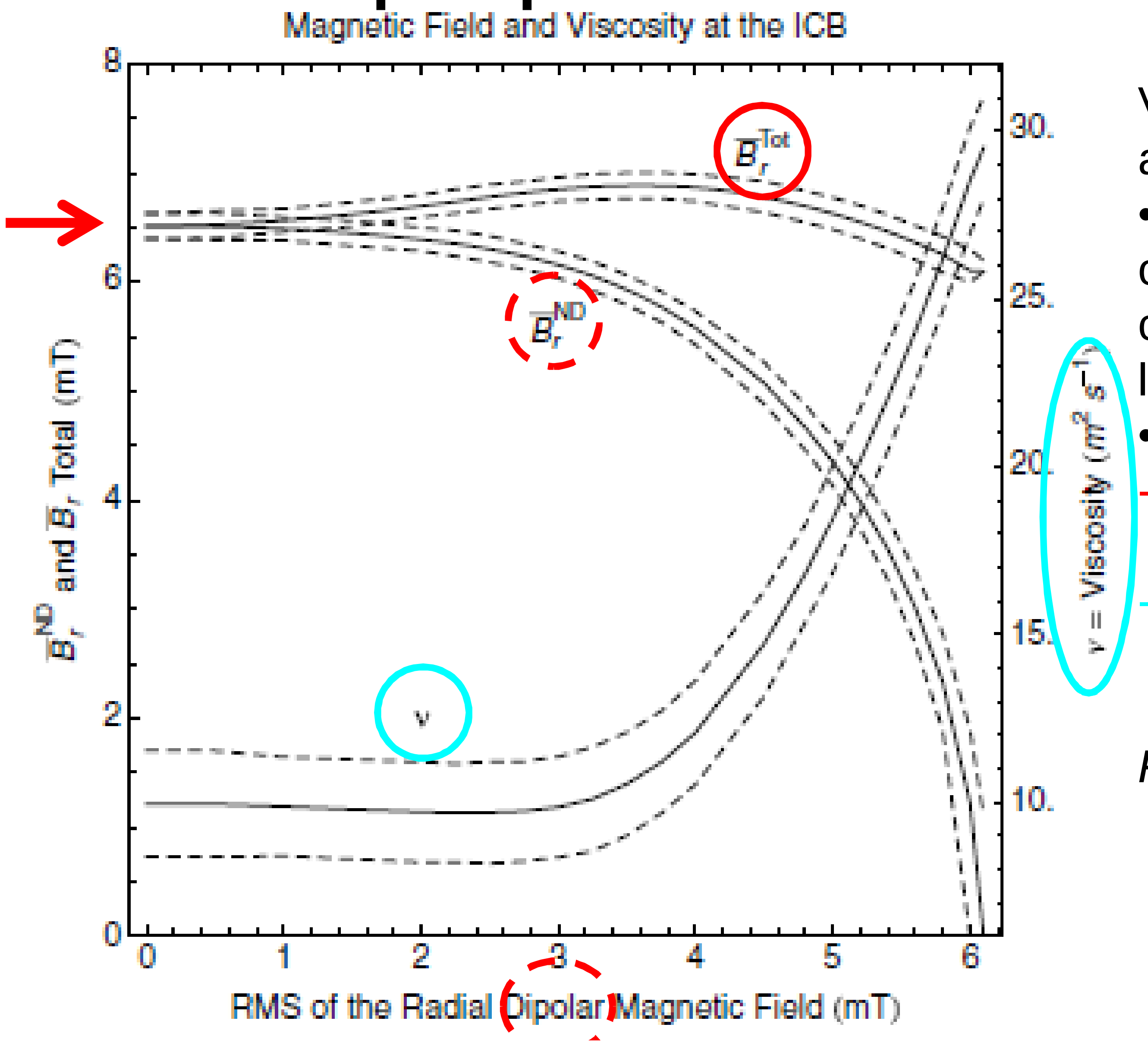
- For EM coupling only: RMS of the radial magnetic field at the CMB: 0.7 mT or higher.
- Viscomagnetic coupling:
  - Allows for lower values of the magnetic field at the CMB.
  - Allows for lower values of mantle conductivity.
  - Outer core viscosity: $\sim 10^{-2}$ m^2 s^{-1}.
    - ➡ Very high value, unlikely to be realistic.

ROB

Constraints on the physical properties of the CMB

- For realistic values of the outer core viscosity, the viscous coupling is negligible.
- Magnetic coupling:
  - Lowermost mantle very high conductivity: 5 10^5 Sm^{-1} (conductivity of iron at core condition) and RMS of the radial magnetic field at the CMB: 0.7 mT.
  - Or more RMS...
- Magnetic field surface observations (degrees <13): RMS ~0.3 mT
- But smaller scales unknown.
- Nutation suggest that most of the energy of the magnetic field at the CMB comes from these!

Constraints on the physical properties of the ICB



Visco-magnetic coupling at the ICB

- Electrical conductivity of the outer and inner cores: known from laboratory experiments.
- Unknown parameters:
  - **Magnetic field at the ICB**
  - **Viscosity of the outer core at the ICB**

From Koot et al. 2010

ROB

Constraints on the physical properties of the ICB

- No solution for a purely EM coupling.
- Outer core viscosity: ~ 10 m^2 s^{-1}: **unrealistic!**
- RMS of the mag. field at the ICB: 6-7 mT.

Another mechanism is required to explain the observed damping of the FICN mode !

Inner core viscous deformation?

Koot & Dumberry EPSL (2011)

– For Inner core viscosity: ~2-7 10^{14} Pa s.

– RMS of the mag. field at the ICB: 4.5 - 6.5 mT

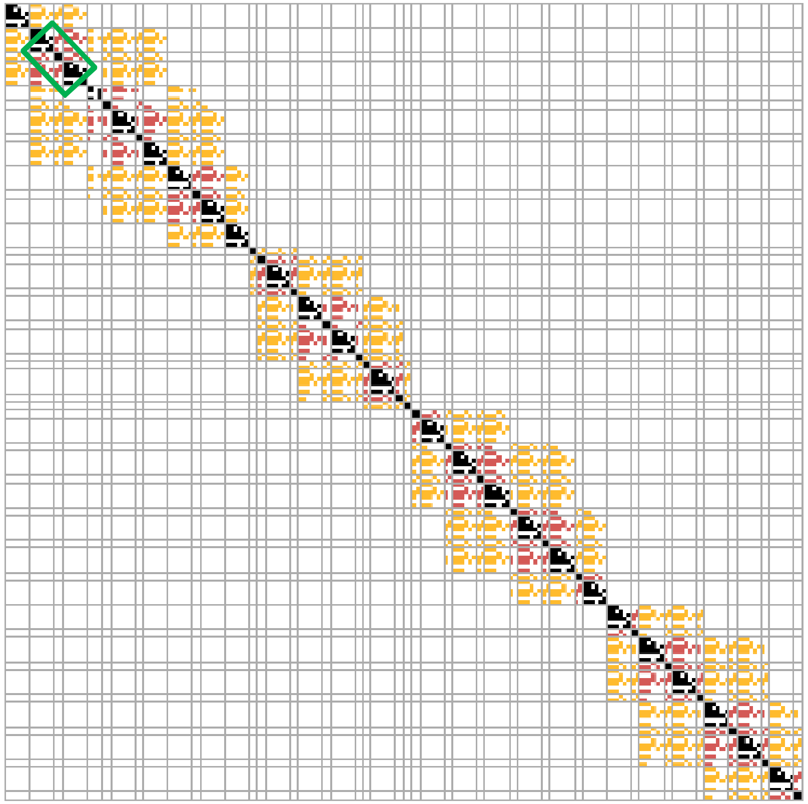
ROB

Modelling the Earth's rotation

- Interior ——— **Realistic** Model ——→ Rotation
- Current model IAU2000 (Mathews et al. 2002)
  - **interior** properties summarised in a set of **parameters**
  - poorly known parameters are **estimated**, **improving** knowledge of the Earth's **interior** (Koot et al. 2010)
  - other parameters are **computed** for a **spherical** Earth
- Former model IAU1980 (Wahr 1981)
  - **full** consideration of the **polar flattening**
  - disregarded **non-hydrostaticity** which affects the **FCN period** (Gwinn et al. 1986), eventually **discarded**
  - since then **refined** (e.g. Huang et al. 2011), now working on **non-hydrostaticity** and associated **triaxiality**

ROB

Non-hydrostaticity & Triaxiality

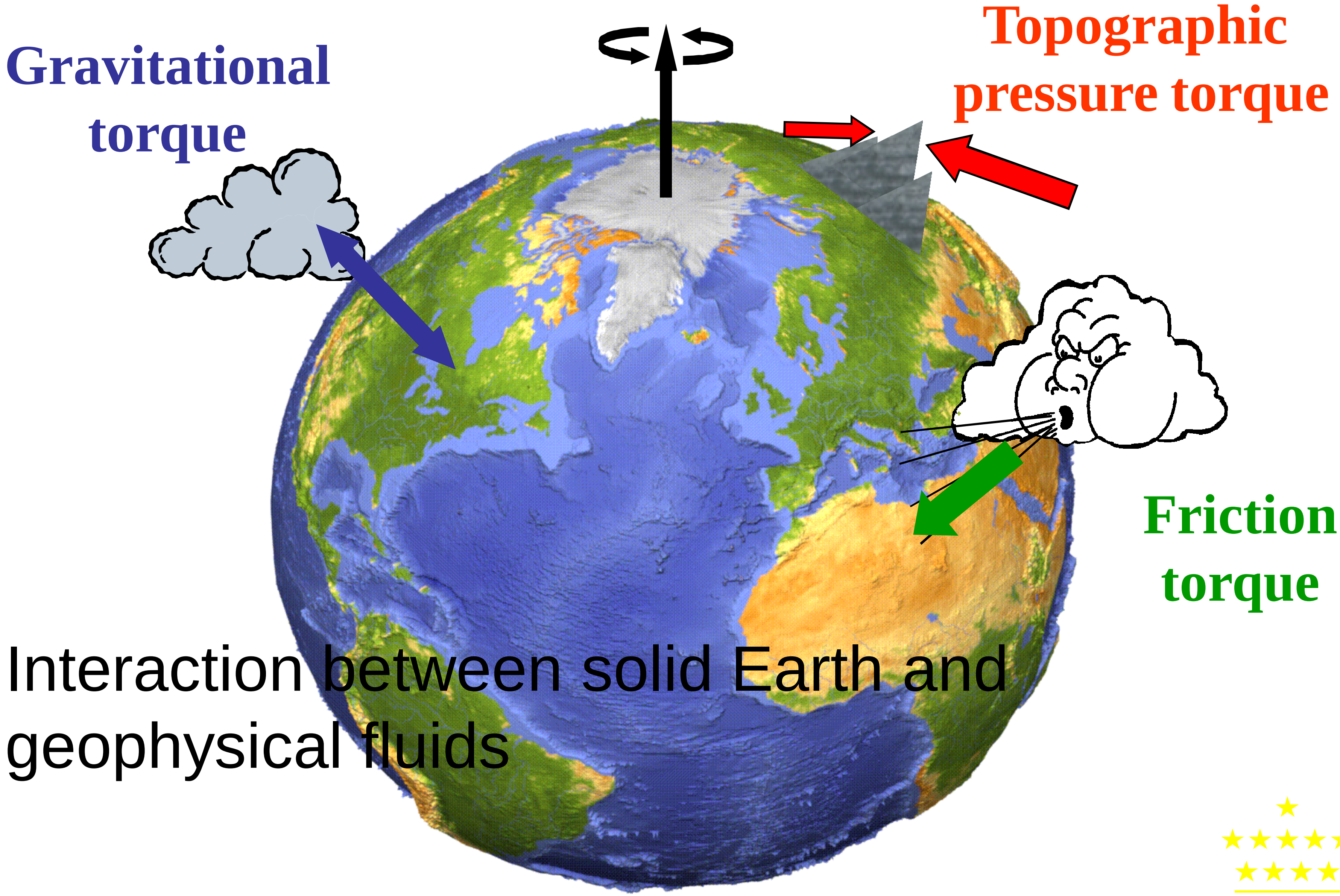


- **Spectral** analysis of the equations of **continuum mechanics**
- Rotation perturbations modelled as infinitesimal **toroidal degree-1** displacement
- ODE submatrix:
  - **spherical**, non-rotating
  - **biaxial**, rotating
  - **triaxial**, rotating, convecting

Work of Antony Trinh

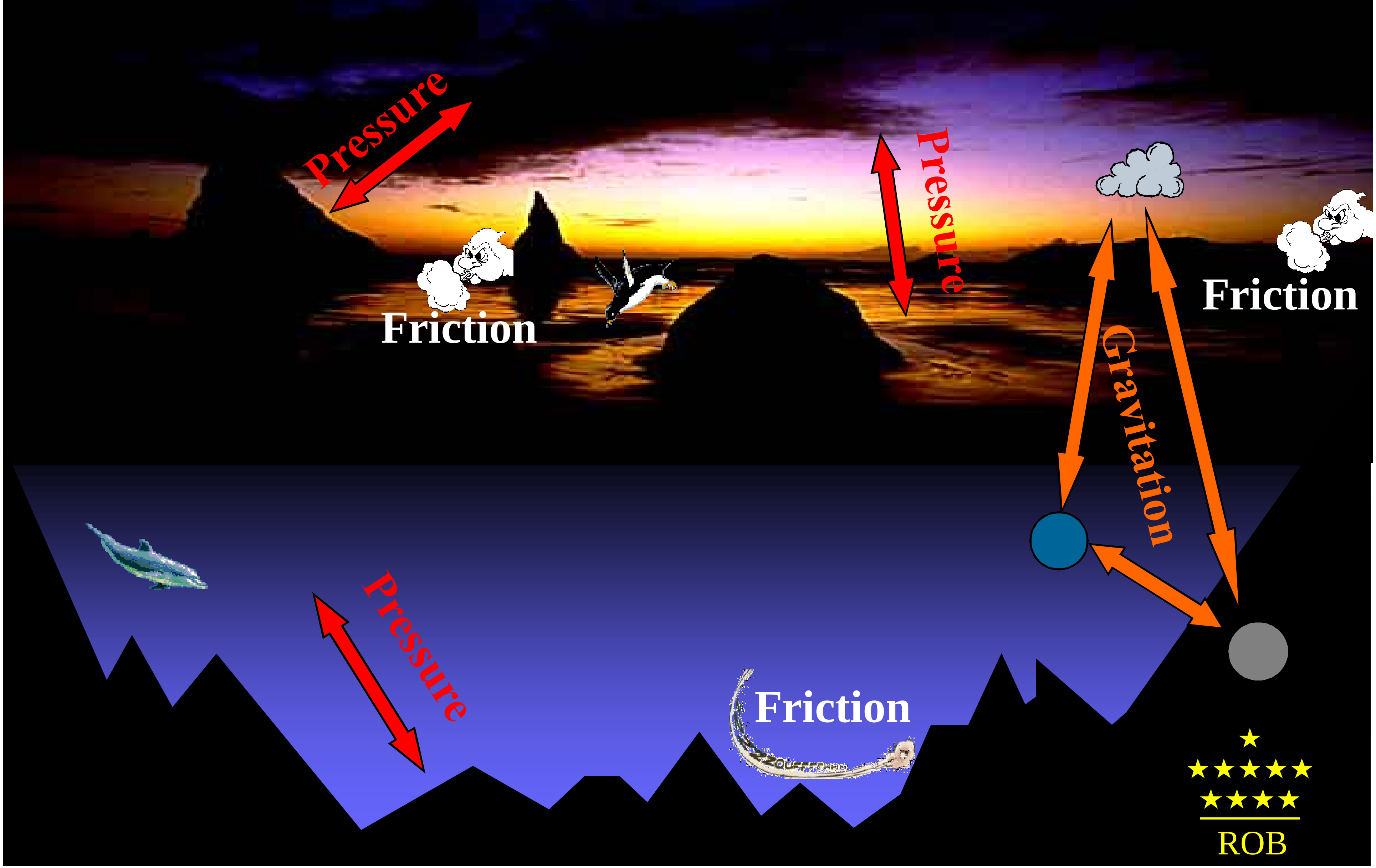
ROB

Angular momentum exchange

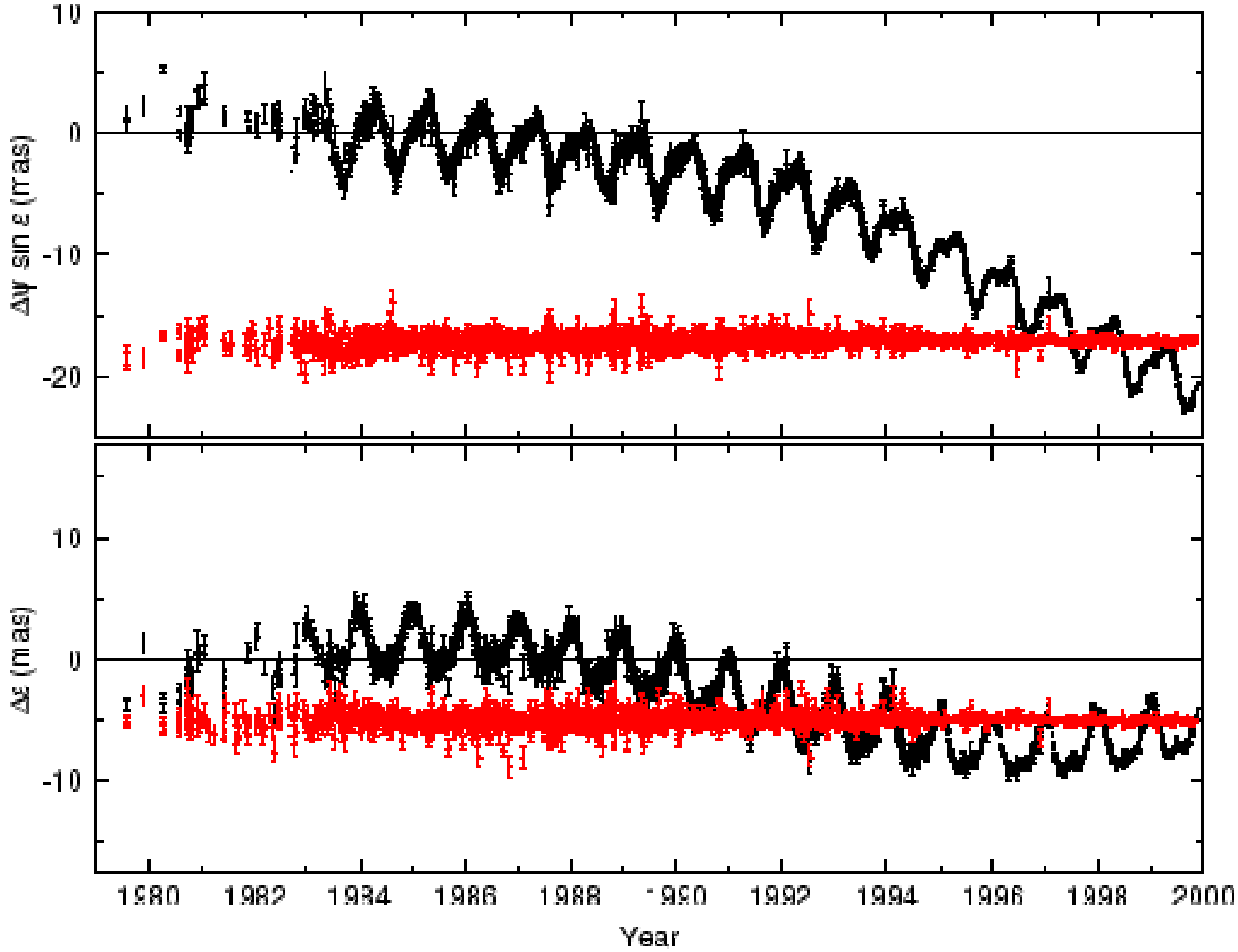


Interaction between solid Earth and geophysical fluids

Earth/atmosphere/ocean interactions

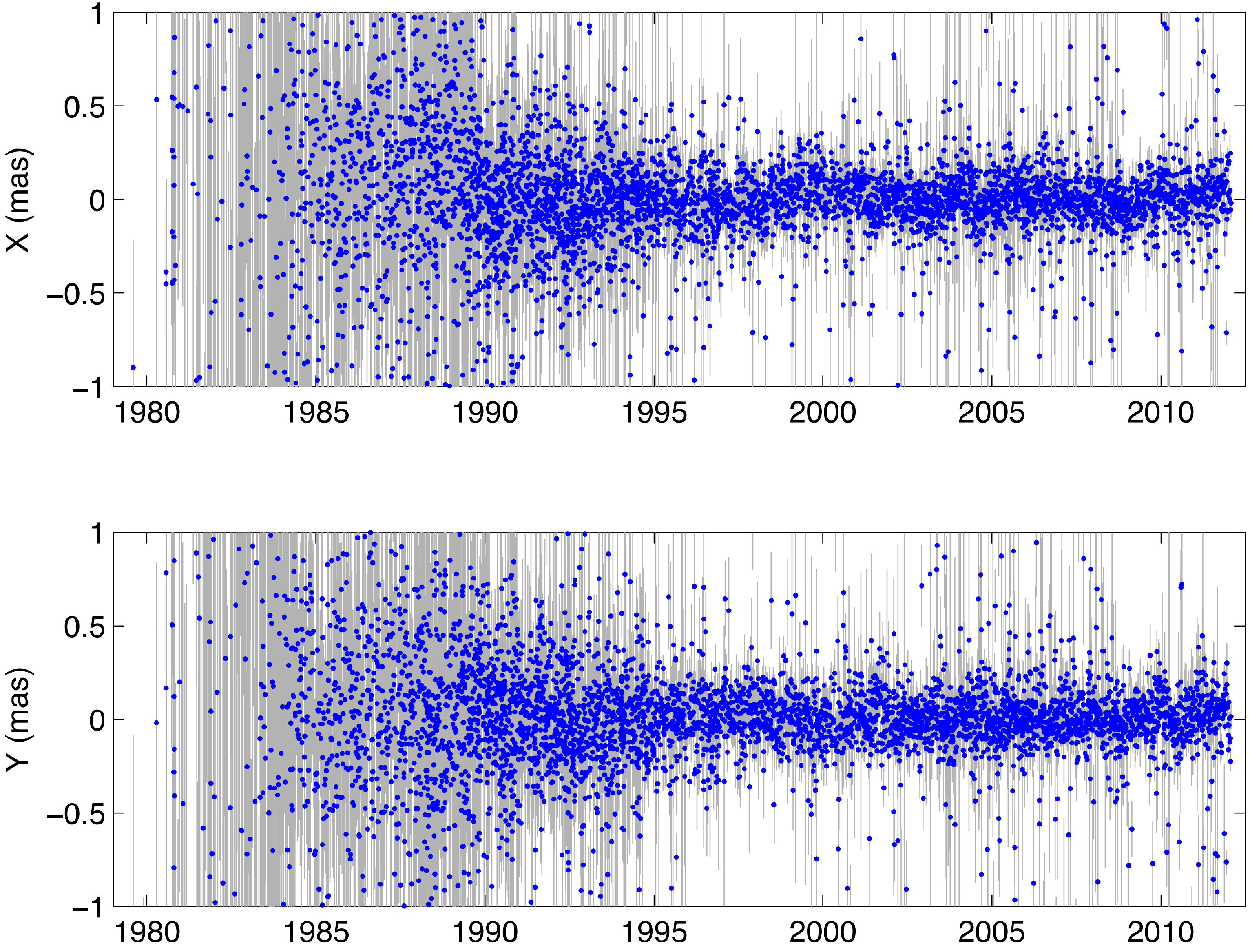


Nutation residuals: observation-model (VLBI-IAU2000A)

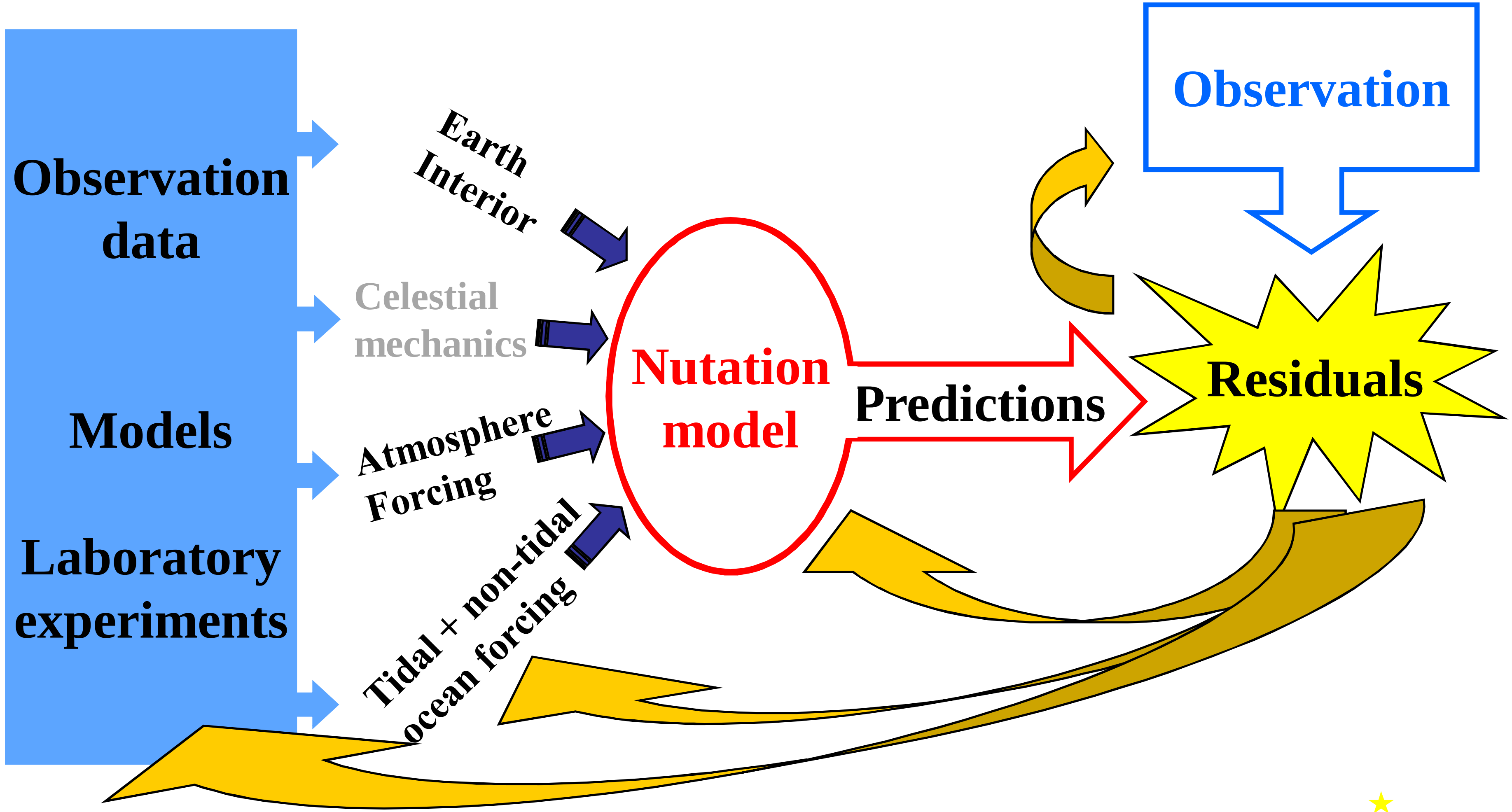


ROB

The nutation after removal of the FCN and main tidal terms



ROB



Better understanding of the Earth interior!

ROB