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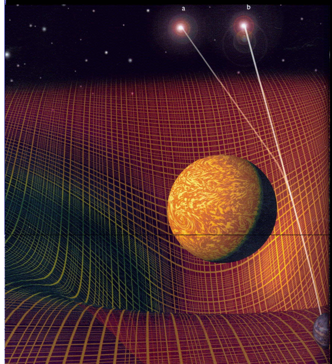
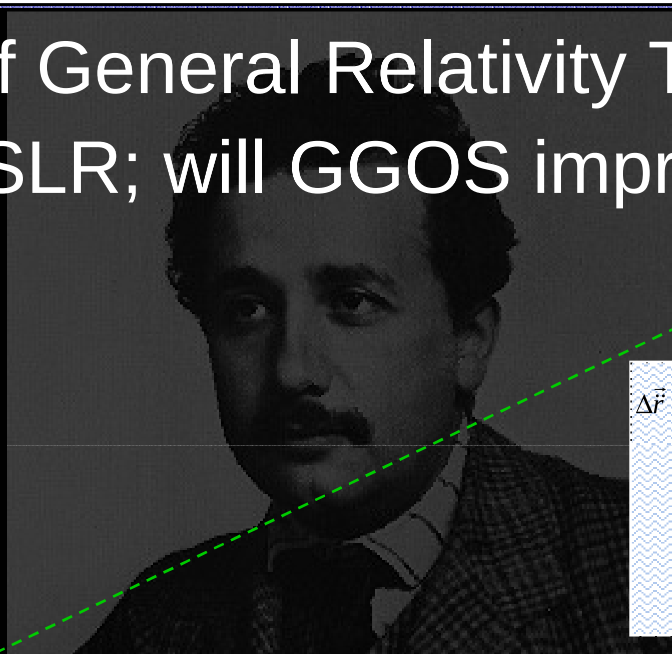
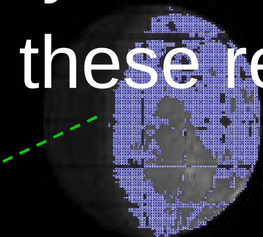
A comparison of General Relativity Theory evaluations using VLBI and SLR; will GGOS improve these results ?

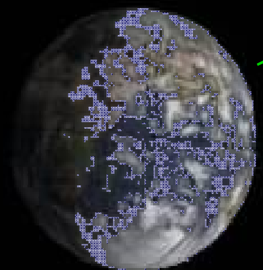






$$\Delta\vec{r} = \frac{GM_\oplus}{c^2r^3}\left\{\left[2(\beta+\gamma)\frac{GM_\oplus}{r} - \lambda(\vec{r}\cdot\vec{r})\right]\vec{r} + 2(1+\gamma)(\vec{r}\cdot\vec{r})\vec{r}\right\} + (1+\gamma)\frac{GM_\oplus}{c^2r^3}\left[\frac{3}{r^2}(\vec{r}\times\vec{r})(\vec{r}\cdot\vec{J}) + (\vec{r}\times\vec{J})\right] + \left\{(1+2\gamma)\left[\vec{R}\times\left(\frac{-GM_s\vec{R}}{c^2R^3}\right)\right]\times\vec{r}\right\},$$

$$\delta\theta = \frac{1}{2}(1+\gamma)\left(4Gm_\odot / c^2 d\right)\left[(1+\cos\Phi)/2\right],$$



$$\tau_{grav} = (1+\gamma)\cdot\frac{GM}{c^3}\cdot\ln\left[\frac{\left|\vec{X}_1\right| + \vec{X}_1\cdot\vec{k}}{\left|\vec{X}_2\right| + \vec{X}_2\cdot\vec{k}}\right],$$

Ludwig Combrinck

Space Geodesy Programme
Hartebeesthoek Radio Astronomy Observatory

ludwig@hartrao.ac.za

7th IVS General Meeting
"Launching the Next-Generation IVS Network"

Madrid (Spain), March 4-9 2012

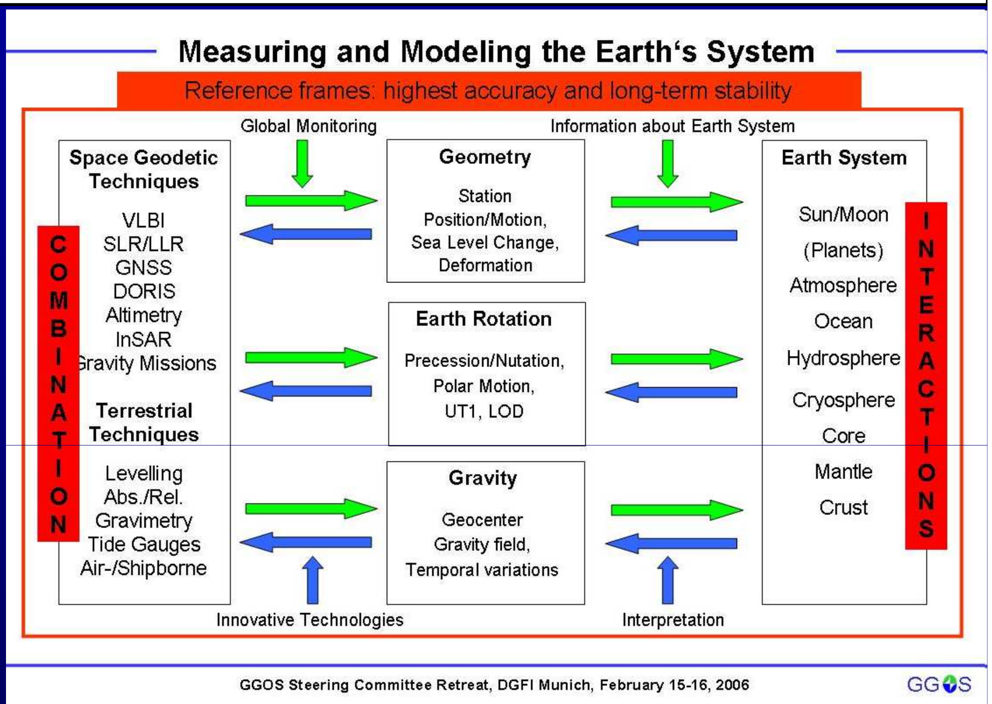
Expected GGOS impact on systems and deliverables

Improved:

- **Network geometry, instrumentation (VLBI, GNSS, SLR/LLR, DORIS)**
- **Additional geophysical instrument collocation (gravimeter, accelerometer, seismometer) leading to ‘tuned’ site specific Love numbers**

- **Improvements in ITRF, ICRF, EOP**
- **Improved models (gravity fields, earth and pole tides, ocean and atmospheric loading, precise orbit determination etc.)**
- **This *should* lead to an improvement in the evaluation of GRT using Space Geodesy techniques, in particular VLBI, SLR and LLR.**

- **Also required are improved GRT delay models, from ~1ps to ~ 0.3psor less for both VLBI and S/LLR (i.e sub-mm accuracy) e.g. developing post post-Newtonian models**



Grace provides improved global gravity field models

GRT implications for space geodetic techniques

GRT effect	VLBI	GPS	SLR
Additional contribution due to Earth's oblateness, amplitude of periodic effect		38 ps Peak to peak (~ 76 ns)	
Additional contribution due to tidal potentials of Sun and Moon		Amplitude of periodic effect 1 ps	
Sagnac effect, depends on path and direction travelled (rotating frame of reference)		Onboard clock runs fast or slow relative to a clock on the geoid Maximum effect 136 ns	
Maximum effect for a stationary GPS receiver located on the geoid		133 ns	
Residual periodic effect at GPS orbital eccentricity of 0.02		Amplitude of residual periodic effect (~ 46 ns) (Keplerian) Peak to peak (~ 19 mm) non-Keplerian <1.2 ns	
Shapiro delay	From 17 × 10^4 ps for a light ray grazing the Sun's limb to ~17 ps when the direction to the source is opposite to the Sun. Delay due to Earth;s gravity field ~ 21 ps for a baseline of 6000 km	~ 19 mm	~ 7 mm
Gravitational light deflection	1.75 arc-sec at limb of Sun (8.5 μrad)		

[redacted]

Why test GRT?

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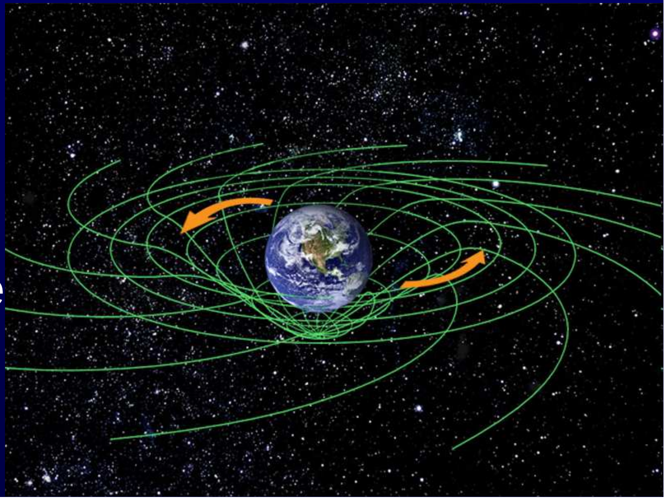
Alternative (*scalar-tensor*) theories, predict small deviations from GR values at a level of: $|\gamma - 1| \approx 10^{-6} - 10^{-7}$ **VLBI and SLR (or any other technique) are not quite there yet.**

- The strength of gravity is given by Newton's (scalar theory) gravitational constant G, it is important to evaluate possible change in G with time (causing an evolving scale of the solar system), current limit on variation of G is given by LLR as:

$$\frac{\dot{G}}{G} = (4 \pm 9) \times 10^{-13}\,\mathrm{yr}^{-1}.$$

Williams, J.G., Turyshev, S.G. and Boggs, D.H. (2004) Progress in Lunar Laser Ranging Tests of Relativistic Gravity, Phys. Rev. Lett. 93:261101, 2004, (arXiv:gr-qc/0411113v2)

- Measurement of space-time curvature and gravitational delay
- Verification of Einstein's equivalence principle
- Measurement of frame dragging
- Relativistic precession of orbits (geodetic (de Sitter) precession)
- Evaluation of PPN parameters Beta (nonlinearity in superposition of gravity) and Gamma (amount of space curvature produced by unit test mass)



Equivalence Principle

- A violation of the Equivalence Principle would lead to the Earth and Moon falling at different rates toward the Sun.

- If the Equivalence Principle is violated, the lunar orbit will move along the Earth-Sun line, which will be seen in a range signature having a 29.53 day synodic period (not the same as the lunar orbit period of 27 days).

- The SEP parameter η is related to the PPN parameters through the Equivalence Principle parameter η where $\eta = 4\beta - \gamma - 3.$

- However PPN parameter γ is conveniently used from other high level estimates

- Current LLR limit (Williams et al. 2004) on the Strong Equivalence Principle:

$$\eta = 4\beta - \gamma - 3 = (4.4 \pm 4.5) \times 10^{-4}$$

- [1] Williams, J.G., Turyshev, S.G. and Boggs, D.H. (2009) Lunar Laser Ranging Tests of the Equivalence Principle with the Earth and Moon, Int. J. Mod. Phys. D18, pp. 1129-1175, 2009, (arXiv:gr-qc/0507083v2)
- [2]Williams, J.G., Turyshev, S.G. and Boggs, D.H. (2004) Progress in Lunar Laser Ranging Tests of Relativistic Gravity, Phys. Rev. Lett. 93:261101, 2004, (arXiv:gr-qc/0411113v2)

Estimation of PPN parameters

- γ indicates how much spacetime curvature is produced per unit mass,
- β indicates how nonlinear gravity is.
- $\gamma = \beta = 1$ in General Relativity.
- Currently the best limits of γ come from measurements of the gravitational time delay of light (Shapiro effect). Radiometric measurements to the Cassini spacecraft set the current limit on γ: $(\gamma - 1) = (2.1 \pm 2.3) \times 10\text{-}5$
- This γ combined with LLR data provides the best limit on β: $(\beta - 1) = (1.2 \pm 1.1) \times 10\text{-}4$.

[1] Williams, J.G., Turyshev, S.G. and Boggs, D.H. (2004) Progress in Lunar Laser Ranging Tests of Relativistic Gravity, Phys. Rev. Lett. 93:261101, 2004, (arXiv:gr-qc/0411113v2)

[2] Bertotti, B., Iess, L. and Tortora, P. (2003) A test of general relativity using radio links with the Cassini spacecraft, Nature 425, pp. 374-376

Technique sensitivity to model and observational errors

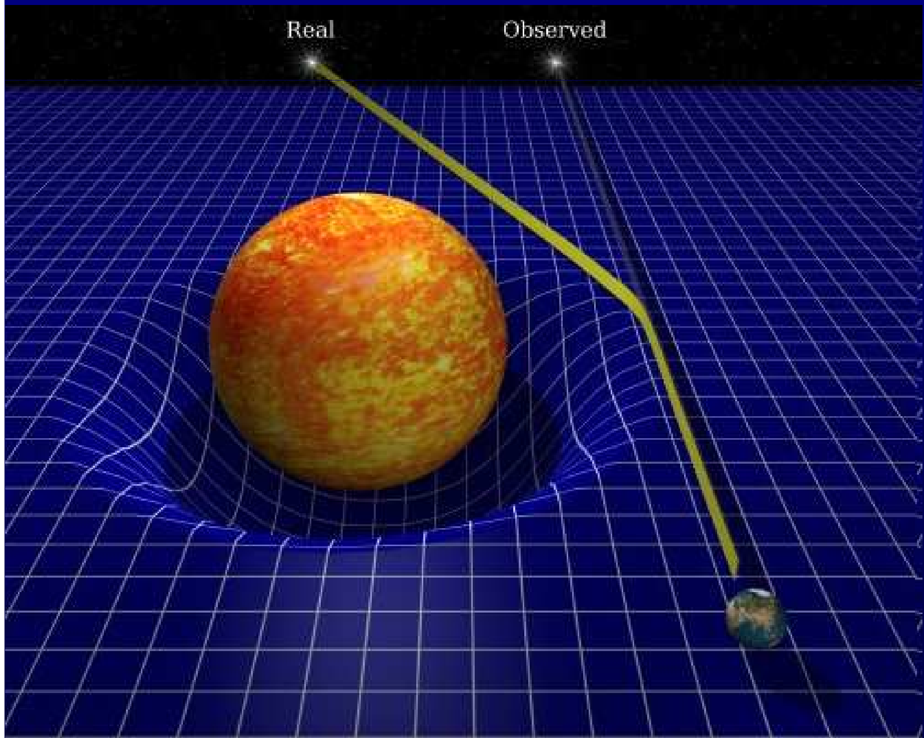
- VLBI GRT estimates are sensitive to:
- intrinsic source structure,
- contribution to delay of the wet neutral atmosphere
- uneven North-South distribution of the VLBI network
- solar coronal plasma for smaller Sun elongations produces large path-length changes (Heinkelmann and Schuh, 2009)

- SLR GRT estimates are sensitive to:
- gravity field model errors in even zonal coefficients (J2, J4, J6..)
- orbit perturbation (model) errors
- contribution to delay of the atmosphere (need to incorporate azimuth dependent components)
- GRT can be embedded in gravity field models
- weak network geometry, particularly in Southern Hemisphere
- only some satellites are suitable for GRT tests, LAGEOS 1, 2 and LARES (recently launched)

- LLR GRT estimates are sensitive to:
- sparsity of network (nothing in Southern Hemisphere)
- very limited data quantity (extremely difficult to range to the Moon, very low return rate)
- depth signature effect in lunar reflector arrays for single photon returns

Tests using VLBI

- Currently PPN γ can be estimated as a solve-for parameter in a VLBI global solution with a precision of 1·10-4 (Lambert and Le Poncin-Lafitte 2009, 2011)



Determination of PPN parameter γ using VLBI

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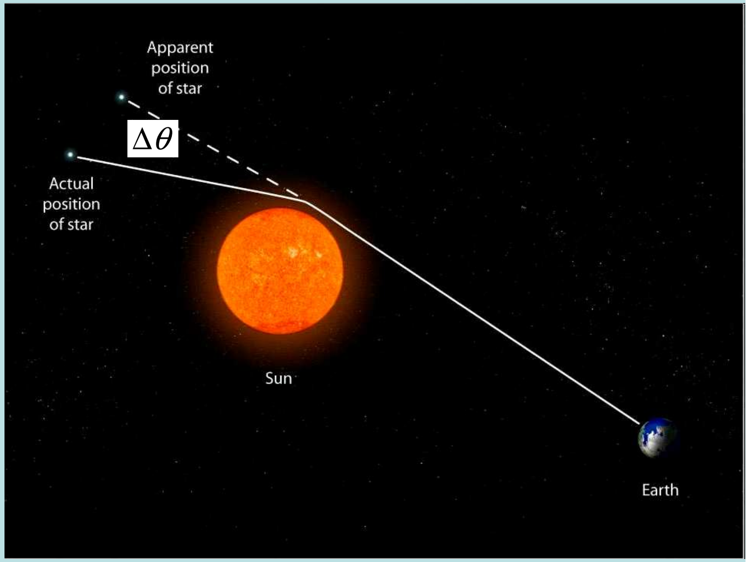
1) An electromagnetic signal (ray of light or radio signal from VLBI source) passing close to the Sun at distance d will be deflected by an angle

$$\delta\theta = \frac{1}{2}(1+\gamma)\left(4Gm_{\odot} / c^2 d\right)\left[\left(1+\cos\Phi\right)/2\right],$$

where the mass of the Sun is denoted by $m_{\odot}$ and Φ is the angle formed between the direction of the incoming electromagnetic signal and the line between Earth and the Sun, d the minimal distance of the ray to the centre of mass of the Sun



2) The partial derivative of the delay relative to γ can be written as

$$\frac{\partial\tau}{\partial\gamma} = \frac{GM}{c^3}\cdot\ln\left[\frac{\left|\vec{X}_1\right| + \vec{X}_1\cdot\vec{k}}{\left|\vec{X}_2\right| + \vec{X}_2\cdot\vec{k}}\right],$$

for estimation of γ utilising the Shapiro delay in a least-squares process.

SLR applications in Relativity

Direct estimation of PPN parameters are possible
Force models used in SLR data processing need to be very accurate

$$\Delta\ddot{\vec{r}} = \frac{GM_{\oplus}}{c^2 r^3}\left\{\left[2(\beta+\gamma)\frac{GM_{\oplus}}{r} - \lambda\left(\dot{\vec{r}}\cdot\dot{\vec{r}}\right)\right]\vec{r} + 2(1+\gamma)\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right\} +$$

$$(1+\gamma)\frac{GM_{\oplus}}{c^2 r^3}\left[\frac{3}{r^2}\left(\vec{r}\times\dot{\vec{r}}\right)\left(\vec{r}\cdot\vec{J}\right) + \left(\dot{\vec{r}}\times\vec{J}\right)\right] +$$

$$\left\{(1+2\gamma)\left[\dot{\vec{R}}\times\left(\frac{-GM_s\vec{R}}{c^2 R^3}\right)\right]\times\dot{\vec{r}}\right\},$$

Validation of Post Newtonian Parameters, Gamma and Beta, □ 8.5×10^{-4} and □ 1.5×10^{-3}
(Combrinck, 2011)

SOUTH AFRICAN JOURNAL OF GEOLOGY, 2011, VOLUME 114.2 PAGE 549-560

Time delay (Shapiro effect)

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- Direct method also allows the solved for Gamma to be passed back into the leasts squares process through the SLR range adjustment:

$$NPR_i = \left(\frac{NPtof_i}{1 \times 10^{12}} \times c \right) \Big/ 2 - \Delta a_i + \Delta CoM_i - \Delta R_{b_i} - \Delta GR_i - \Delta \varepsilon_i$$

$$t_2 - t_1 = \frac{\left| \vec{x}_2(t_2) - \vec{x}_1(t_1) \right|}{c} + \sum_J \frac{(1+\gamma) GM_J}{c^3} \ln \left(\frac{r_{J1} + r_{J2} + \rho}{r_{J1} + r_{J2} - \rho} \right)$$

GRT components of acceleration

- Gamma and Beta are determined through solving for the acceleration in a leasts squares adjustment

- The partial derivatives are passed to the sensitivity matrix as part of the rigorous linearisation of the orbit trajectory, together with the various parameters that determine the various forces affecting the satellite orbit.

- State vector is estimated every 24 hours

$$\Delta\ddot{\vec{r}} = \frac{GM_\oplus}{c^2 r^3}\left\{\left[2(\beta+\gamma)\frac{GM_\oplus}{r} - \gamma\left(\dot{\vec{r}}\cdot\dot{\vec{r}}\right)\right]\vec{r} + 2(1+\gamma)\left(\vec{r}\cdot\dot{\vec{r}}\right)\dot{\vec{r}}\right\} +$$

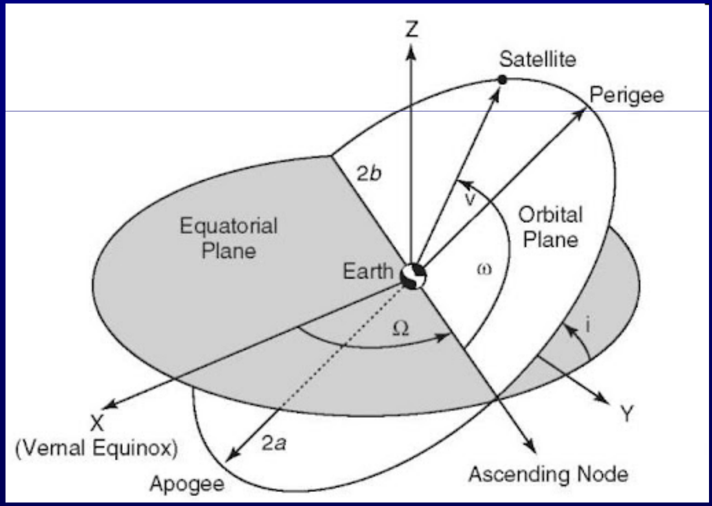
$$(1+\gamma)\frac{GM_\oplus}{c^2 r^3}\left[\frac{3}{r^2}\left(\vec{r}\times\dot{\vec{r}}\right)\left(\vec{r}\cdot\vec{J}\right) + \left(\dot{\vec{r}}\times\vec{J}\right)\right] +$$

$$\left\{(1+2\gamma)\left[\dot{\vec{R}}\times\left(\frac{-GM_s\vec{R}}{c^2 R^3}\right)\right]\times\dot{\vec{r}}\right\},$$

$$\ddot{\vec{r}}_{GRT} = \ddot{\vec{r}}_{Schwarzschild} + \ddot{\vec{r}}_{Lense-Thirring} + \ddot{\vec{r}}_{deSitter}$$

Frame dragging estimates using SLR

- Initially proposed by Cugusi and Proverbio (1977)
- First reported results, were by Ciufolini et al. (1996) who analysed the SLR range observations of satellites LAGEOS and LAGEOS II utilising the software package GEODYN II, both nodes of LAGEOS I and II were used as well as the argument of perigee of LAGEOS II, accuracy ~ 30%
- Subsequent estimates used only the nodes (Ciufolini et al. 2004; Ciufolini et al. 2006) in a 'butterfly' configuration of the retrograde LAGEOS I and the prograde LAGEOS II orbits, accuracy 5-10%



- Estimation of perigee shift in the Schwarzschild gravitoelectric and gravitomagnetic field by Lucchesi and Peron (2010), placed new constraints on non-Newtonian gravity. Models for general relativity were not included in the orbit determination, thereby obtaining the relativistic signal in the residuals. Utilising LAGEOS II pericenter residuals they were able to obtain a 99.8% agreement with the predictions of Einstein's theory.

Extrapolation of VLBI results

Authors	Standard error
Counselman et al. (1974)	±0.06
Fomalont and Sramek (1975)	±0.022
Fomalont and Sramek (1976)	±0.018
Robertson and Carter (1984)	±0.005
Carter, Robertson and MacKay (1985)	±0.003
Robertson, Carter and Dillinger (1991)	±0.002
Lebach et al. (1995)	±0.0017
Eubanks et al. (1997)	±0.00031
Shapiro et al. (2004)	±0.00021
Lambert and Le Poncin-Lafitte (2009)	±0.000152

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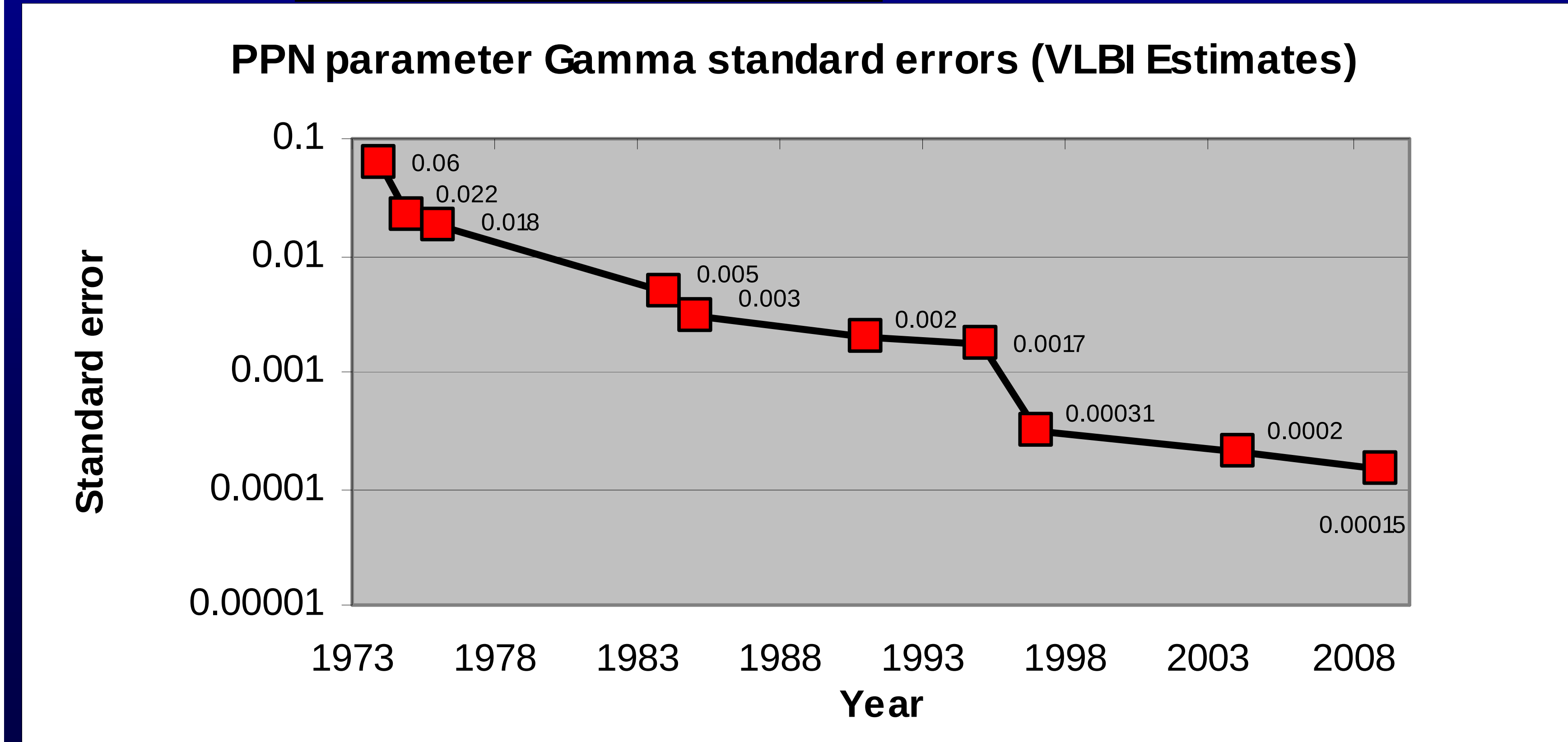
Standard errors associated with geodetic VLBI evaluations of PPN parameters. Table from Heinkelmann and Schuh (2009).



Expected VLBI accuracy

- A straight line fit constrained to the first and last estimate provides a value of 2.5×10^{-5} when using the fitted function to predict towards 2020.

- If this predicted accuracy level is achieved by VLBI, supported by the developments around VLBI2010 in the GGOS framework it would be better/comparable to the accuracy (currently the best) of the estimate of (5.2×10^{-5}) achieved (Bertotti et al 2003) during the microwave tracking of the Cassini spacecraft on its approach to Saturn.

- With dedicated strategies and projects aimed at testing GRT, this value of 2.5×10^{-5} could be improved (after GGOS implementations) to $\sim1\times10^{-6}$ or better! That is, VLBI will be the most accurate test of γ and will be in the 'hot zone' of supporting/not supporting GRT

- Error estimates of VLBI are very robust and conservative due to large volume of data involved

Some current values and bounds on the PPN Parameters Gamma and Beta (test of GRT)

None meet ‘hot zone’ of testing

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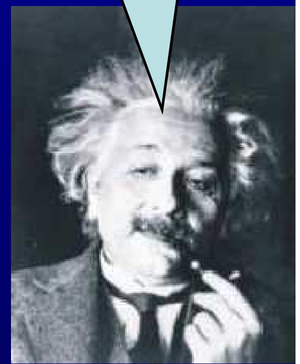
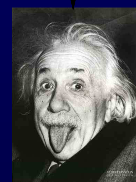
		Value	Bound (1 sigma)	
γ-1 How much space curvature per unit mass?	Time delay	-1.3×10^{-5}	5.2×10^{-5}	Cassini-Earth Sun conjunction microwave tracking (Anderson et al. 2004)
	Light deflection (delay of signal passing the Sun)	-6×10^{-5} 1.6×10^{-4} 8×10^{-5}	3.1×10^{-4} 1.5×10^{-4} 1.2×10^{-4}	Astrometric VLBI (Eubanks et al. 1997) Lambert & Le Poncin-Lafitte, 2009 2011, (includes VLBA sessions, latest models)
	Light deflection (delay of signal passing the Sun)	2×10^{-4}	3×10^{-4}	Standard error not sigma? (Fomalont et al. 2009) VLBI
	Radar observations of planets and spacecraft	-1×10^{-4}	2×10^{-4}	(Pitjeva, 2005)
	GRT Satellite acceleration LAGEOS 1 LAGEOS 2 (Combrinck 2011)	6.5×10^{-4} 9.0×10^{-4}	7.4×10^{-4} 9.6×10^{-4}	GRT components of satellite acceleration + Shapiro delay
β-1 How “non-linear” is gravity?	Radar observations of planets and spacecraft	0.0000	1.0×10^{-4}	(Pitjeva, 2005)
	Light deflection	-1.9×10^{-4}	2.6×10^{-4}	Astrometric VLBI (Eubanks et al. 1997)
	GRT Satellite acceleration LAGEOS 1 LAGEOS 2 (Combrinck 2011)	1.2×10^{-3} 1.4×10^{-3}	1.4×10^{-3} 1.5×10^{-3}	GRT components of satellite acceleration





South Africa's contribution to help getting space geodesy test into the 'hot zone'………..
No LLR currently in Southern Hemisphere

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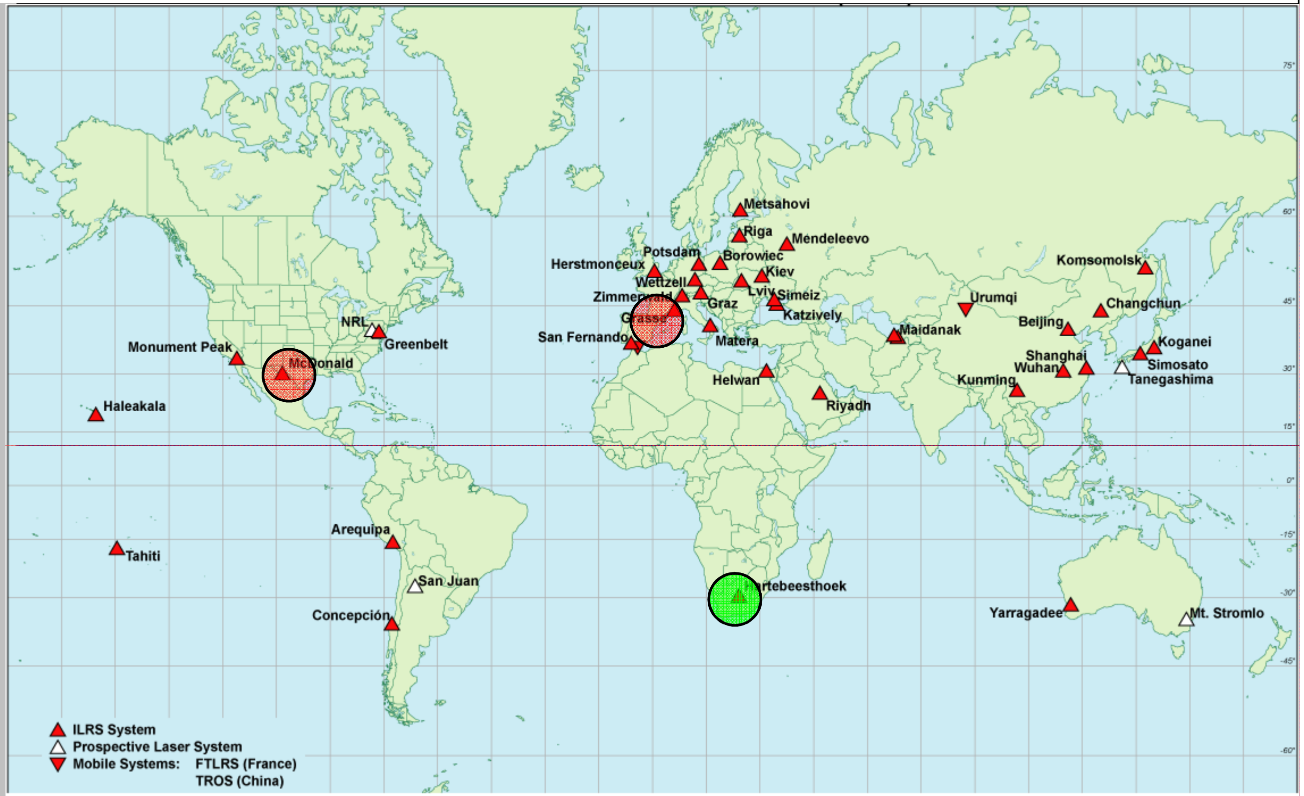
- A Southern Hemisphere LLR will strengthen the geometry of the LLR network and should improve the determination of the orientation of the Moon

- A dual system S/LLR will provide added coverage of SLR data in an area very sparsely covered



Five small Targets, huge distance

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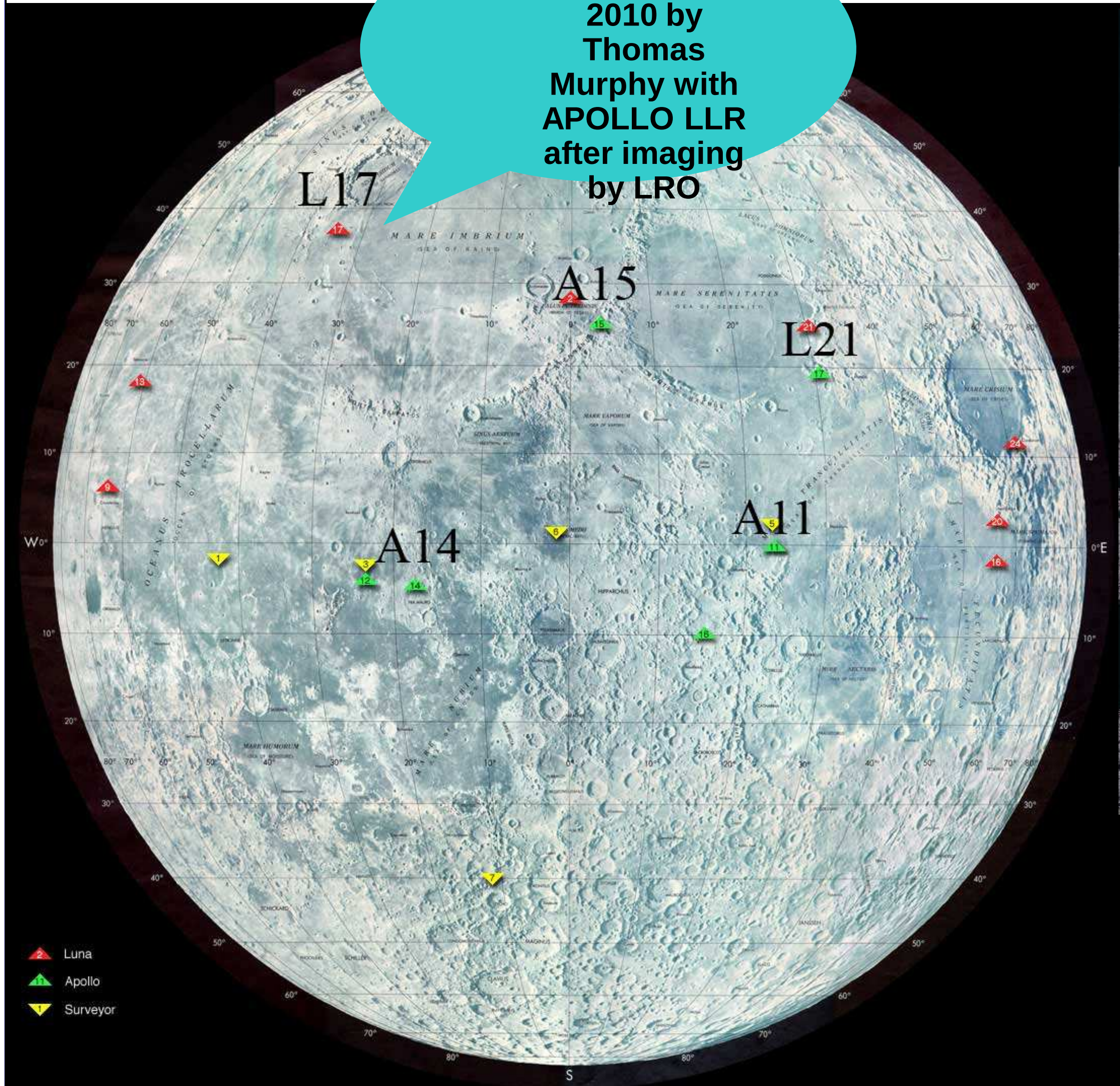
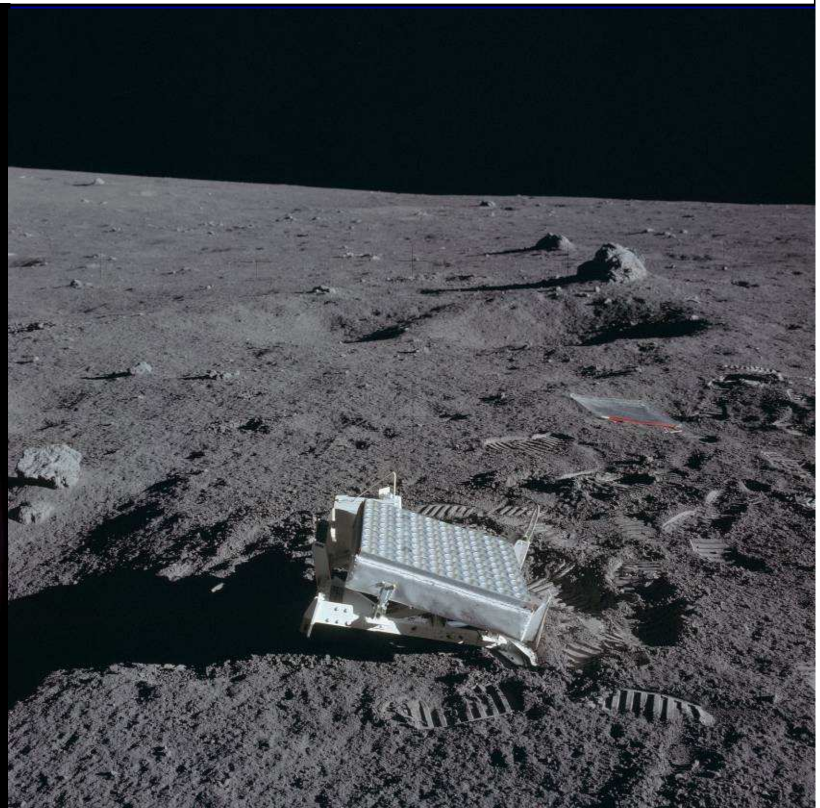
Found on 22 April 2010 by Thomas Murphy with APOLLO LLR after imaging by LRO





- Lunokhod arrays: 14 triangular shaped cubes, each side 11 cm
- Apollo arrays used fused silica “circular opening” cubes, 3.8 cm diameter each
  - Apollo 11 and 14 arrays, 100 cubes
  - Apollo 15, 300 cubes

Ex-Observatoire de la Cote d’Azur 1-m SLR telescope

Ex-OCA 1 m telescope

- Telescope housed in run-off enclosure
- Crane to assist in disassembly and refurbishment
- Totally nuts and bolts construction to facilitate future removal to appropriate site
- Stable and massive foundation for tests

Run-off enclosure on steel tracks

Control centre based on 12 m shipping container

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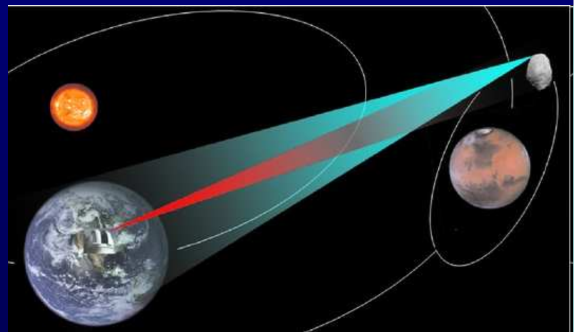
- Located next to MOBLAS-6
- Curved roof to provide shade

Conclusions

- VLBI and S/LLR as geodetic techniques may soon move into the 'hot zone' of testing γ and β to a level where Einstein's GRT may either be re-validated or deviations of its solutions may be detected.
- GGOS will support VLBI, SLR and LLR to improve their validations of GRT, but only to its fullest extent if all aspects of GGOS are addressed, networks, equipment, models, observing strategy and processing strategies.
- Requirements for better results include scheduling of VLBI observations closer to the Sun, construction of VLBI beacon/laser transponder units for placement in orbit and on the Moon+suitable planets



- GGOS currently only has 3 themes…….it would be important not too exclude it's 4th theme…..fundamental physics…. as mm accuracy in ALL the space geodetic techniques and reference frames is finally constrained by our understanding of the geometry of space

Thank you!

Acknowledgements

Funding received from the National Research Foundation (NRF), South Africa, towards this project is appreciated. This work is based on research supported by NRF grant IFR2011041500034.