TECHNISCHE UNIVERSITÄT WIEN
Vienna University of Technology

RESEARCH GROUP ADVANCED GEODESY
Institute of Geodesy and Geophysics

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Solid Earth tide parameters from VLBI measurements and FCN analysis

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Solution characteristics (1)

- Software VieVS
  - Module for global solution allows to estimate TRF and CRF simultaneously, together with other geodetic and geophysical parameters in a common adjustment

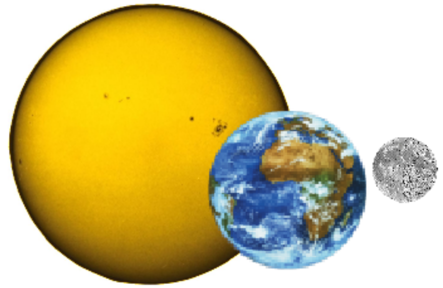
- Data: 3360 24-hour IVS sessions (1984.0 – 2011.0)

- A priori data and models
  - VTRF2008, ICRF2
  - ERP: C04 08, IERS2010 model for subdaily ERP variations applied
  - Precession/nutation model: IAU 2006/2000A
  - Solid Earth tides: IERS2010
  - pole tides: cubic model, IERS2010
  - ocean loading: FES2004
  - atmosphere loading: APL series and S1/S2 tides (Petrov and Boy, 2004)
  - a priori troposphere gradients: DAO
  - mapping functions: VMF1

Love and Shida numbers for diurnal tides

- Love and Shida numbers characterize the site displacement caused by the solid Earth tides

- In the Earth model with fluid core and anelastic mantle Love and Shida numbers of the diurnal tides are frequency dependent and contain imaginary parts



- The differences in the Love and Shida numbers to the constant nominal values cause corrections to the radial and transverse displacement:

$$\delta r_f^{21} = \left(\delta R_f^R \sin(\theta_f + \lambda) + \delta R_f^I \cos(\theta_f + \lambda)\right)\sin(2\varphi)$$

$$\delta \vec{t}_f^{21} = \left(\delta T_f^R \cos(\theta_f + \lambda) - \delta T_f^I \sin(\theta_f + \lambda)\right)\sin(\varphi)\vec{e}$$

$$+ \left(\delta T_f^R \sin(\theta_f + \lambda) + \delta T_f^I \cos(\theta_f + \lambda)\right)\cos(2\varphi)\vec{n}$$

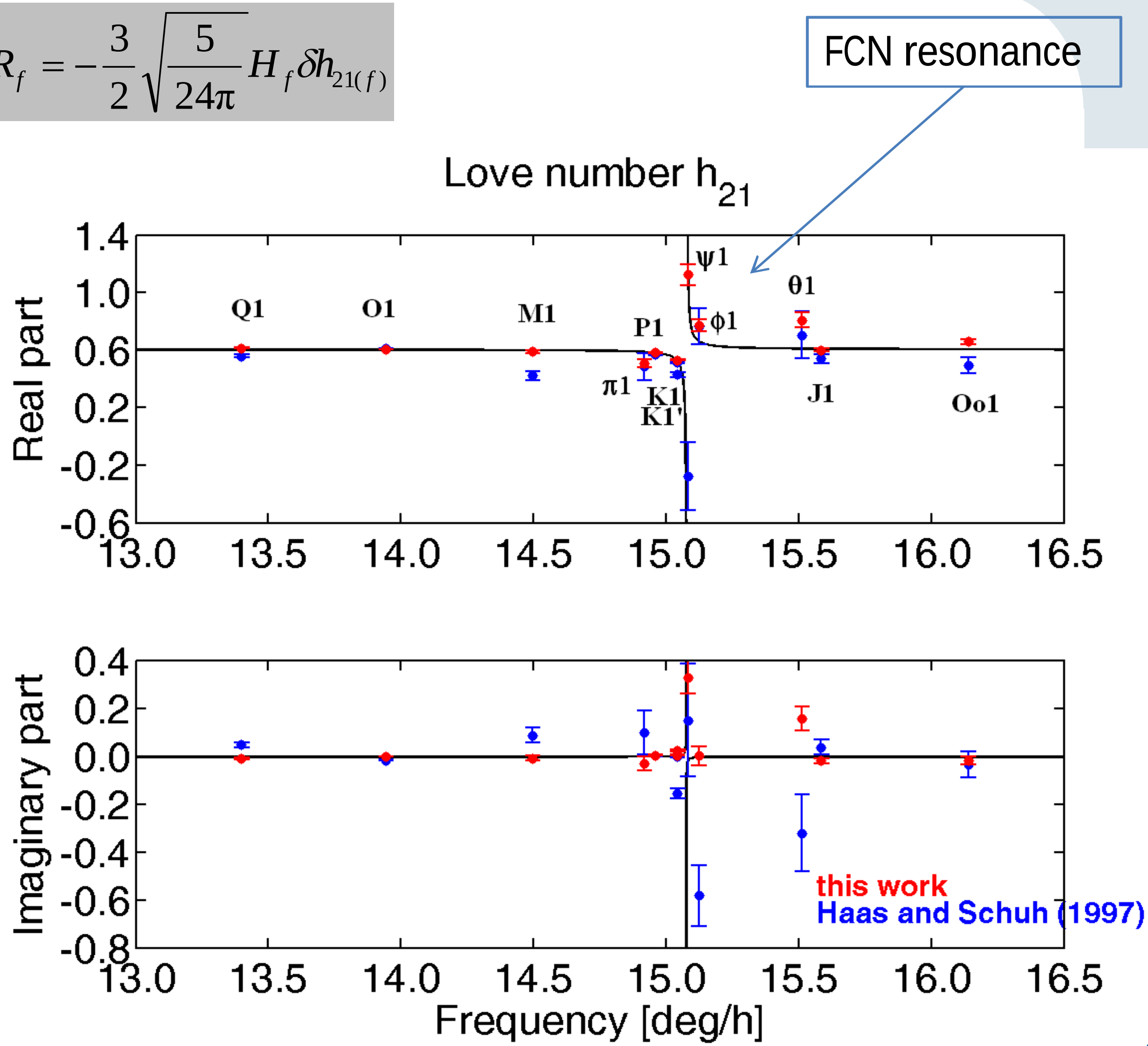
corrections to displacement coming from the harmonic terms of the second degree tidal potential in the diurnal band

$\delta R_f, \delta T_f$	corrections to the nominal amplitudes
φ, λ	latitude and longitude of the station
θ_f	tidal harmonic argument

Love numbers in diurnal band

$$\delta R_f = -\frac{3}{2}\sqrt{\frac{5}{24\pi}} H_f \delta h_{21(f)}$$

Tide	H_f [mm]	ΔR_f^R [mm]
Q1	-50.21	0.22 ± 0.08
O1	-262.25	0.00 ± 0.09
M1	20.62	0.09 ± 0.08
π1	-7.16	-0.22 ± 0.08
P1	-122.35	-0.03 ± 0.08
K1	369.14	-0.08 ± 0.10
K1‘	49.97	-0.16 ± 0.08
ψ1	2.94	-0.09 ± 0.08
ϕ1	5.26	-0.22 ± 0.08
θ1	3.94	-0.30 ± 0.08
J1	20.62	0.09 ± 0.08
Oo1	11.29	-0.23 ± 0.08



Love num

$$\delta R_f = -\frac{3}{2}\sqrt{2}$$

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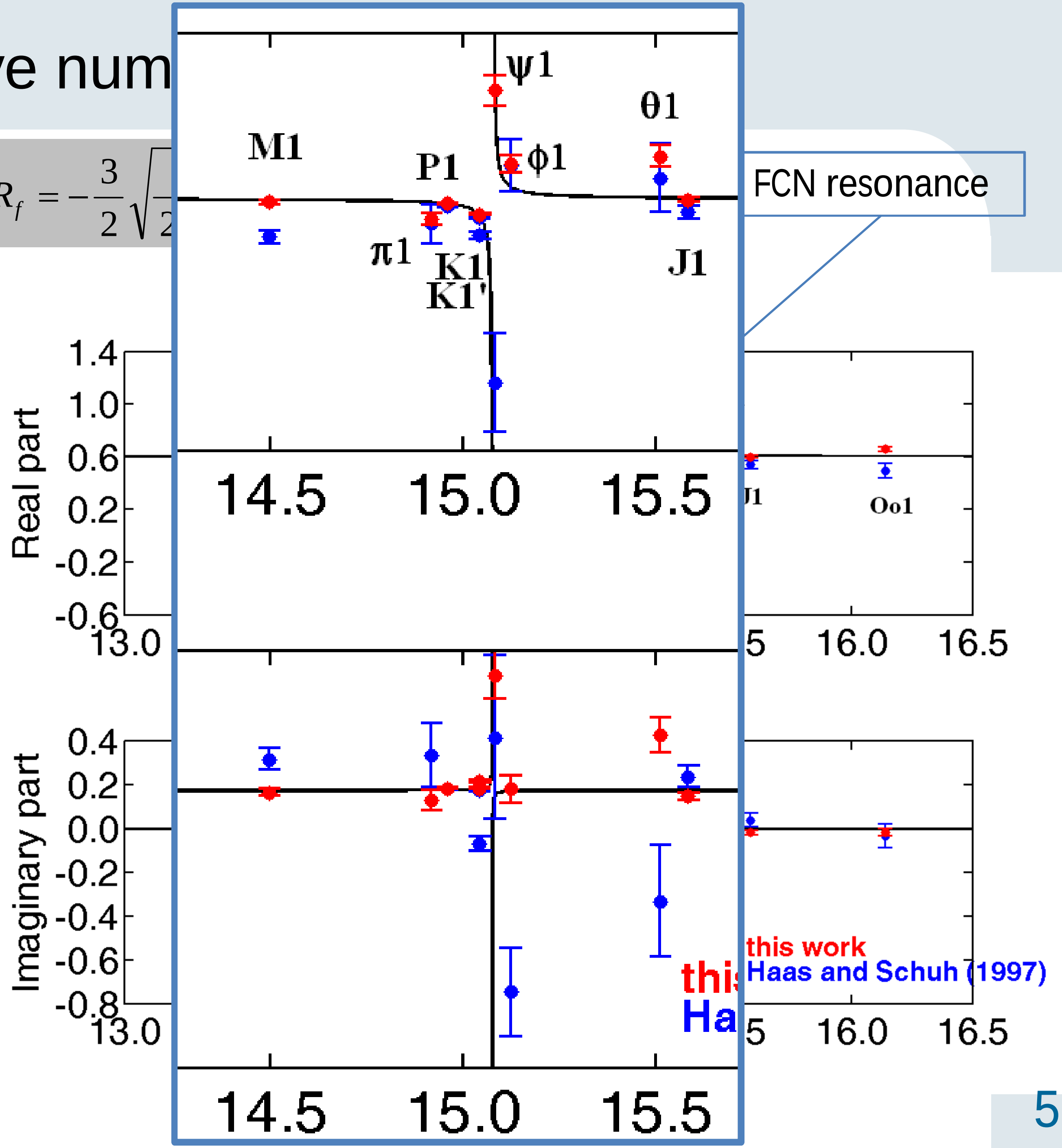
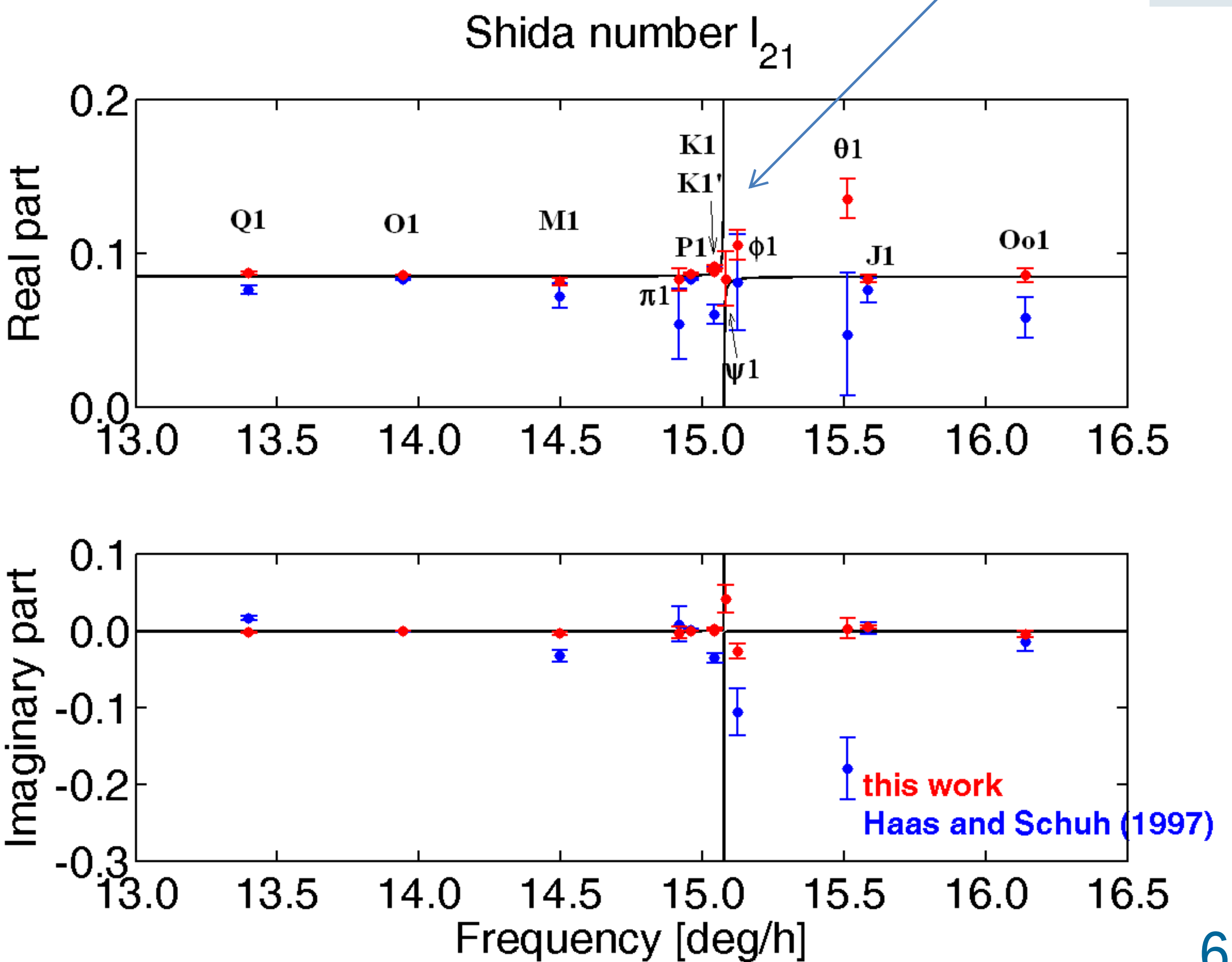


Shida numbers in diurnal band

$$\delta T_f = -3\sqrt{\frac{5}{24\pi}} H_f \delta l_{21(f)}$$

Tide	H_f [mm]	ΔT_f^R [mm]
Q1	-50.21	0.09 ± 0.04
O1	-262.25	0.20 ± 0.04
M1	20.62	0.05 ± 0.04
π1	-7.16	-0.01 ± 0.04
P1	-122.35	-0.08 ± 0.04
K1	369.14	-0.27 ± 0.08
K1‘	49.97	-0.15 ± 0.04
ψ1	2.94	-0.03 ± 0.04
ϕ1	5.26	-0.09 ± 0.04
θ1	3.94	-0.15 ± 0.04
J1	20.62	0.02 ± 0.04
Oo1	11.29	-0.01 ± 0.04

FCN resonance



Shida nu

$\delta T_f = -3\sqrt{}$

Tide	H_f [mm]	ΔT_f^R [mm]
Q1	-50.21	0.09 ± 0.04
O1	-262.25	0.20 ± 0.04
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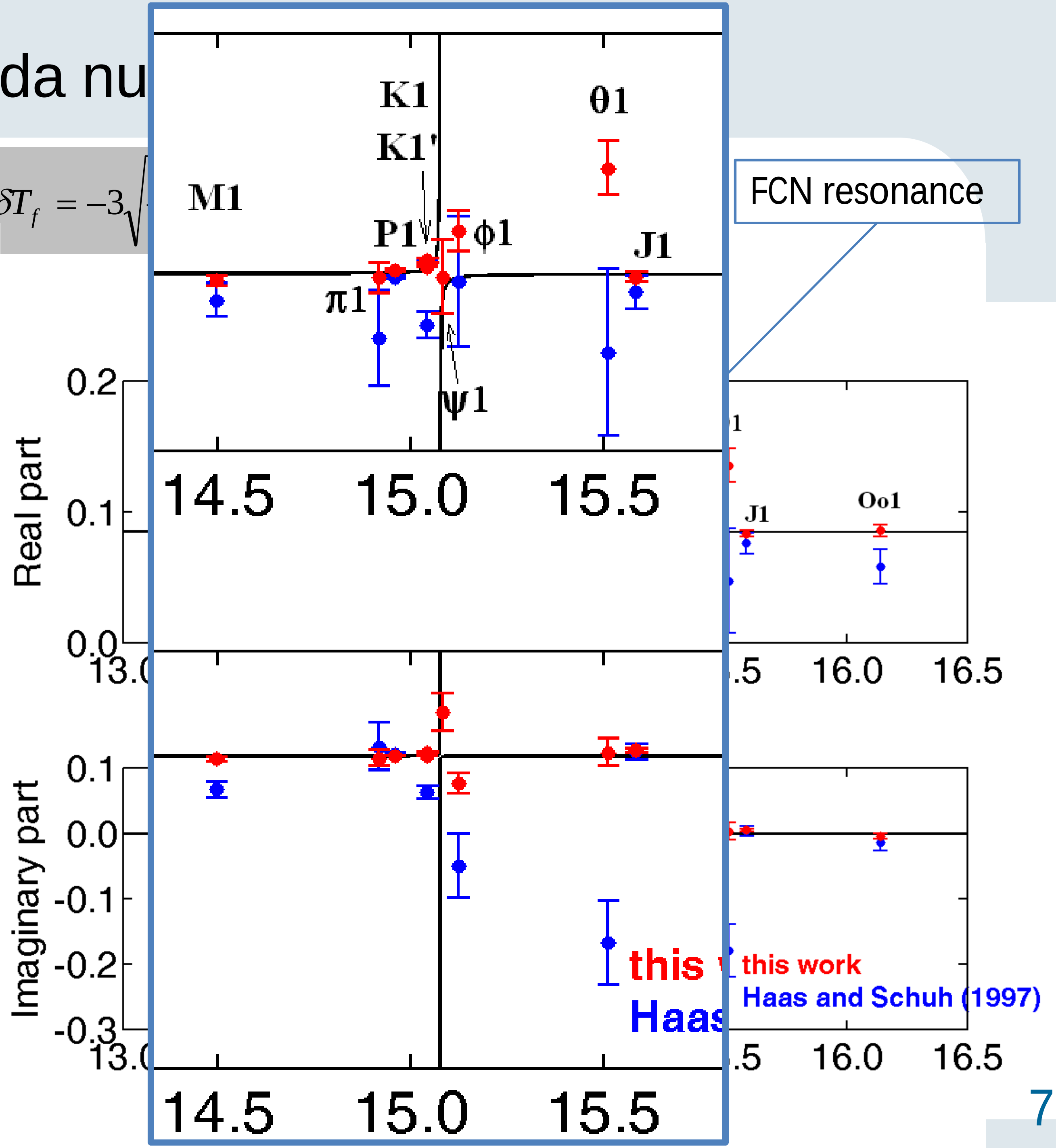
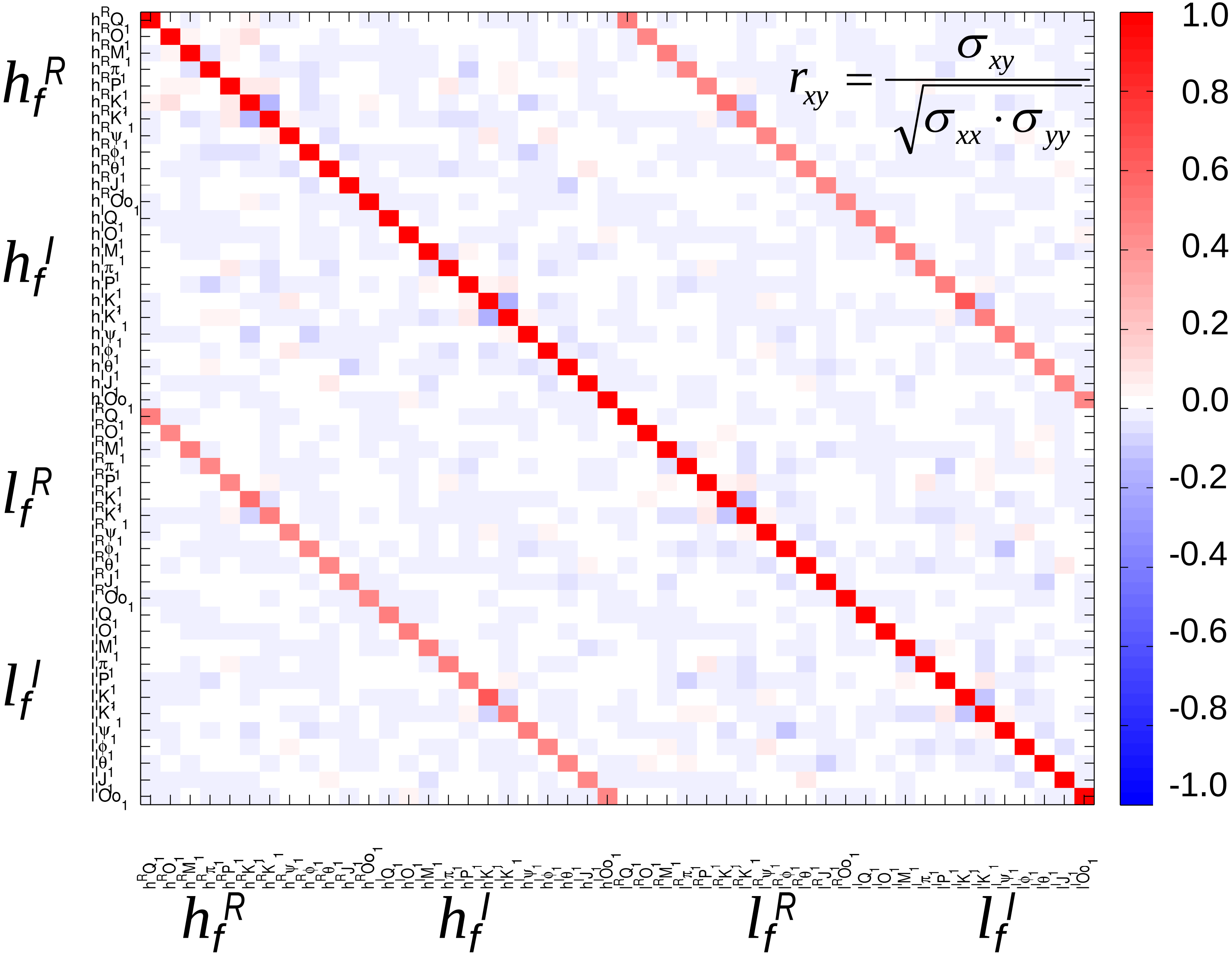


Correlation coefficients



max. correlations are between Love and Shida numbers of the same tide (about 0.45)

each VLBI measurement contains information about radial and also tangential displacement

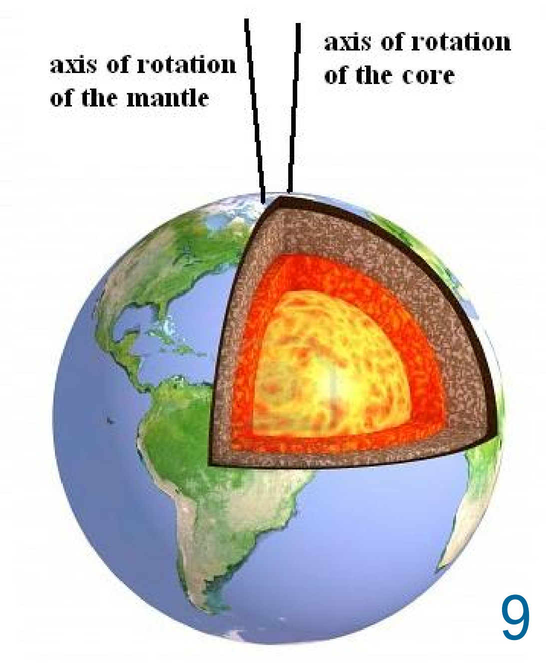
Free Core Nutation

- FCN is caused by the misaligment of the rotational axis of the Earth mantle and the fluid core

  → small periodic motion of the Earth's axis of rotation with a period of about **430 days** in the **celestial** reference frame, or with a **nearly diurnal** period, if observed in the **terrestrial** frame

- In the analysis of VLBI data, the presence of FCN is visible in:
  - the frequency dependence of the Love and Shida numbers in the diurnal band of solid Earth tides
  - the motion of the CIP (celestial intermediate pole) in the celestial reference system

We estimated the FCN period from both phenomena.



FCN period from solid Earth tides

- Resonance formulae for the Love/Shida numbers in the diurnal band (IERS2010)

$$L_f = L_0 + \frac{L_{CW}}{\sigma_f - \sigma_{CW}} + \frac{L_{FCN}}{\sigma_f - \boxed{\sigma_{FCN}}} + \frac{L_{FICN}}{\sigma_f - \sigma_{FICN}}$$

h_{21f}, l_{21f}

Chandler Wobble

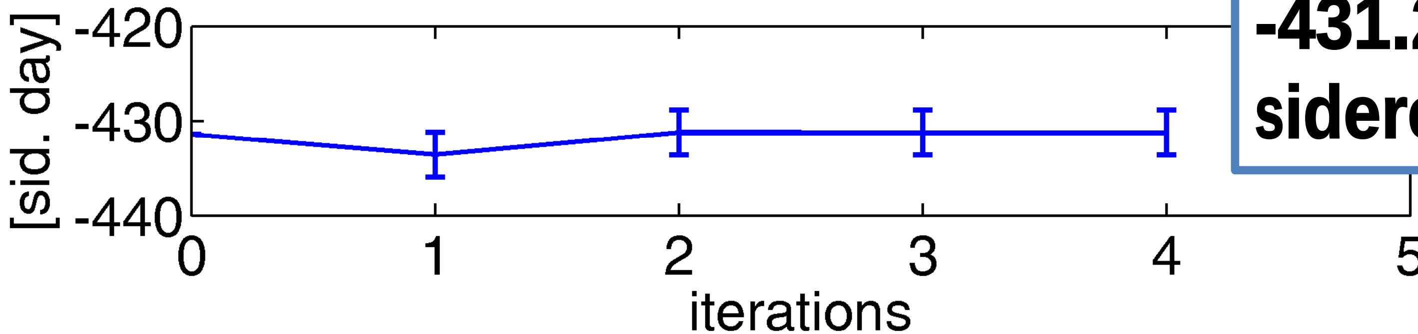
FCN

Free Inner Core Nutation

- Partial derivative of the time delay w.r.t. the FCN frequency (contained in the solid Earth tides)

$$\frac{\partial \tau}{\partial \sigma_{FCN}} = k \cdot QRW \cdot \frac{\partial b}{\partial \sigma_{FCN}}$$

k – source vector
QRW – transformation matrix
b – baseline vector



-431.23 ± 2.4 sidereal days

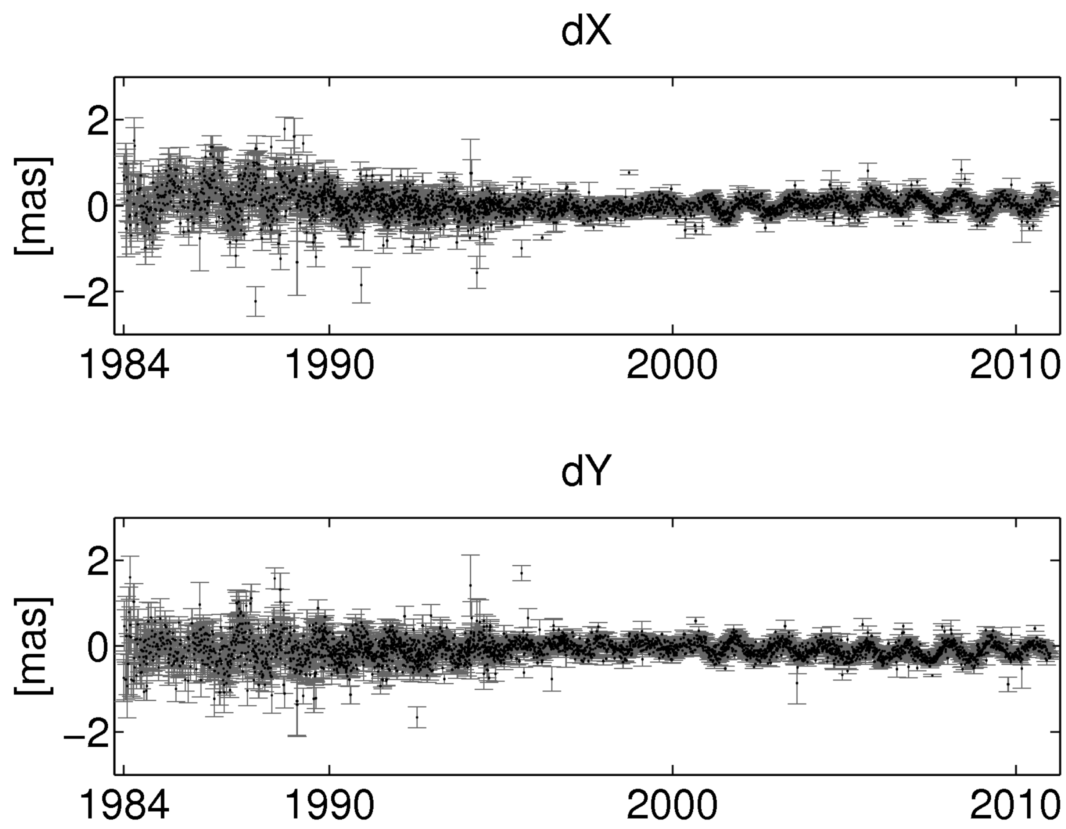
Celestial pole offsets (CPO)

- CPO = residuals to the IAU 2006/2000A precession-nutation model → describes motion of the Earth‘s rotational axis in the celestial system

- FCN is not included in the precession-nutation model, because it cannot be predicted rigorously

our 1st step

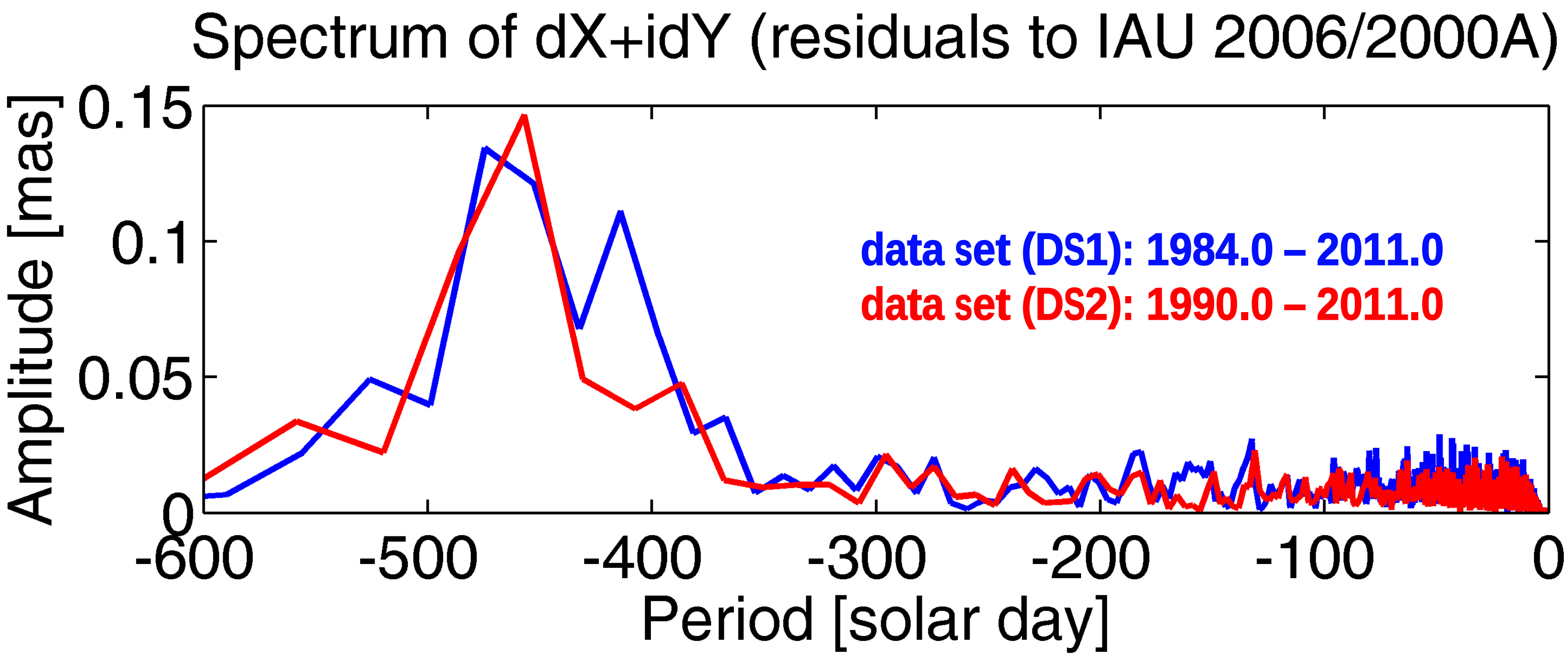
estimation of the FCN period from the CPO by applying a spectral analysis



CPO estimated with VieVS

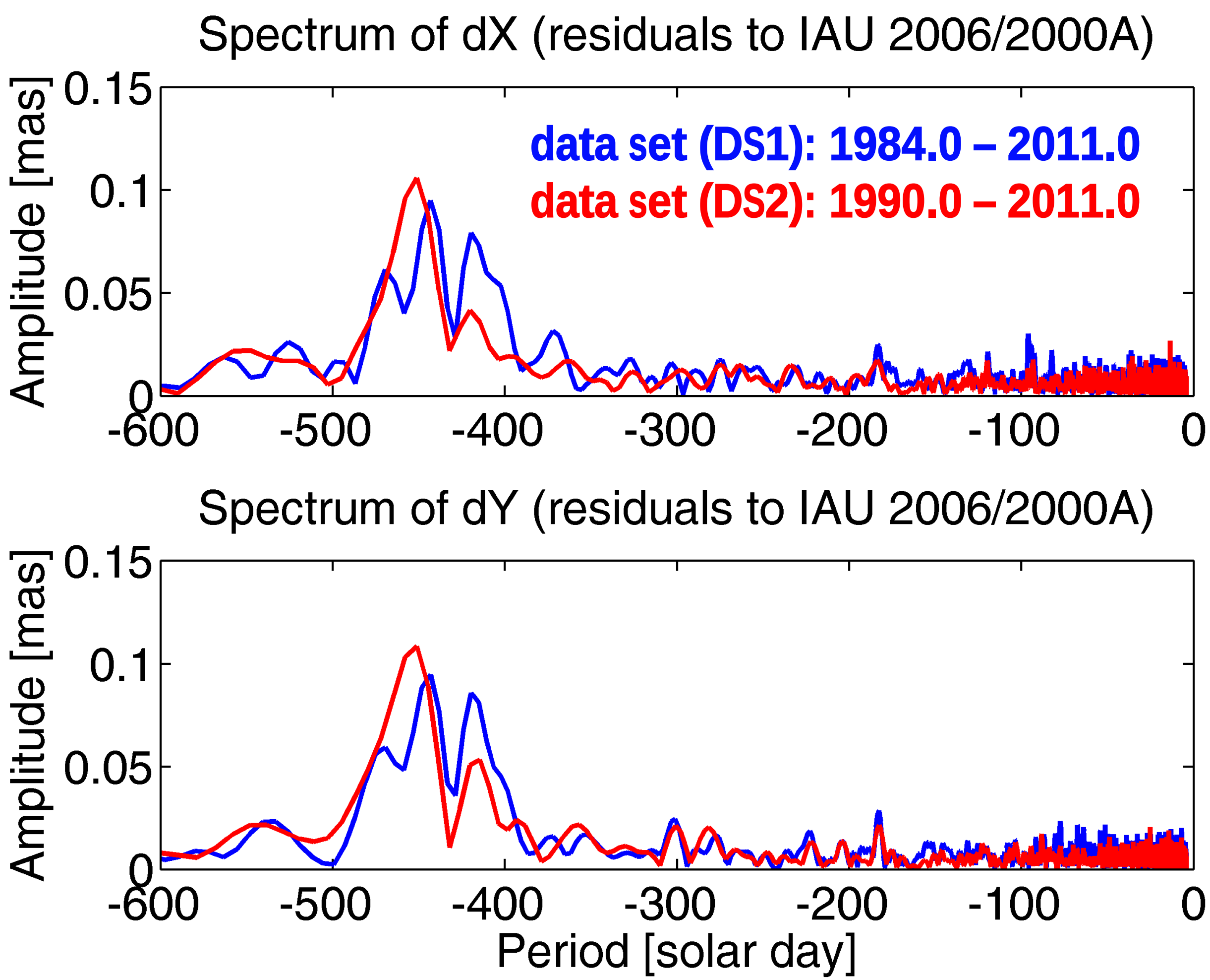
Fast Fourier Transform of the CPO

- FFT was applied to two sets of data (1984 - 2011 and 1990 - 2011)
- data were interpolated to three-days intervals using Lagrange interpolation
- broad peaks appear around the expected FCN period
  - from DS1 – double peak around **-410 days** and **-470 days**
  - from DS2 – one peak around **-460 days**



Spectral analysis with a CLEAN algorithm

- CLEAN algorithm allows to analyze non-equidistantly spaced time series
- dX and dY have to be analyzed separately



- Estimated periods
  - from DS1:
    three peaks between
    -400 and -500 days:
    -415, -451 and about
    -480 solar days
  - from DS2:
    one peak around
    -460 days

- Results of the direct CPO analysis from other authors

  - Schmidt et al. (2005) – analysis by means of wavelet technique using Morlet function (FCN period varied between **-425 to -450 days**)

  - Malkin and Miller (2007) – discrete Fourier spectral analysis (two broad peaks around **-410 and -450 days**)

  - Vondrak et al. (2005) – Fast Fourier Transform (different estimates of FCN period dependent on the time spans of input data sets: **about -425, -430 and -470 days**)

Are the broad double peaks a sign of two harmonic components
or
is there only one wave with a constant period but changing phase?

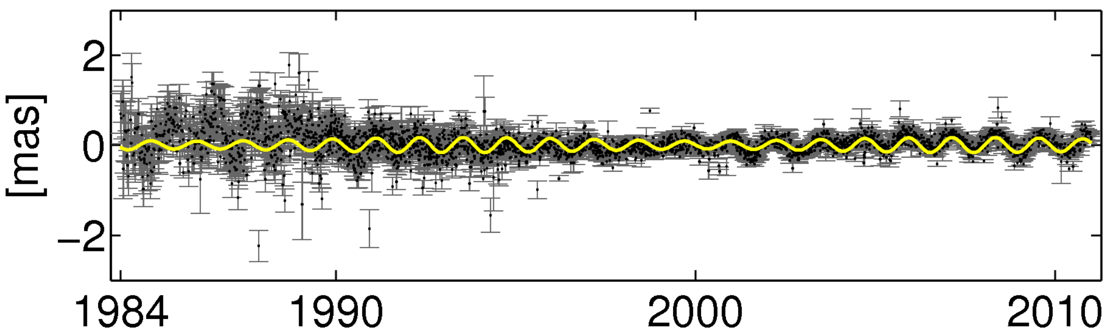
Model for the CPO as a superposition of 2 harmonic waves with constant periods and amplitudes

- we chose two periods: P1 = -415 solar days (from the spectral analysis by CLEAN algorithm)
  P2 = -451 solar days

- constant amplitudes were estimated as global parameters in the adjustment of VLBI data:

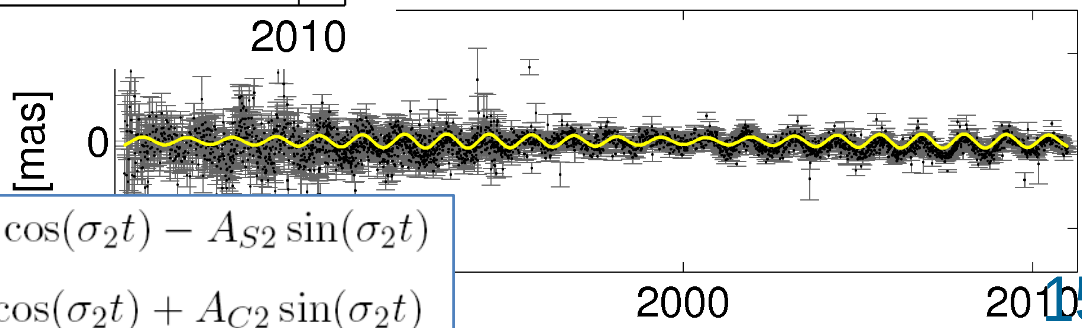
P [solar days]	A_C [mas]	A_S [mas]	A [mas]	Φ [deg]
-415	-0.0120 ± 0.0011	0.0386 ± 0.0011	0.0404 ± 0.0011	107.27 ± 1.81
-451	0.0165 ± 0.0011	-0.1264 ± 0.0011	0.1275 ± 0.0011	-82.56 ± 0.49

dX



Although the geophysical interpretation of such a model is questionable, a good agreement with the residuals to the IAU 2006/2000A model can be seen (after 1990, where the data started to be less noisy).

dY



$$dX = A_{C1}\cos(\sigma_1 t) - A_{S1}\sin(\sigma_1 t) + A_{C2}\cos(\sigma_2 t) - A_{S2}\sin(\sigma_2 t)$$

$$dY = A_{S1}\cos(\sigma_1 t) + A_{C1}\sin(\sigma_1 t) + A_{S2}\cos(\sigma_2 t) + A_{C2}\sin(\sigma_2 t)$$

FCN period as global parameter from the motion of the CIP (1)

- we added the contribution of the FCN to the a priori description of the precession-nutation

$$\boldsymbol{Q}(t) = \boxed{d\boldsymbol{Q}(t)} \cdot \boldsymbol{Q}(t)_{(IAU)} = \boxed{\begin{bmatrix} 1 & 0 & X_{FCN} \\ 0 & 1 & Y_{FCN} \\ -X_{FCN} & -Y_{FCN} & 1 \end{bmatrix}} \cdot \boldsymbol{Q}(t)_{(IAU)}$$

precession-nutation matrix

description of the X_{FCN} and Y_{FCN} follows the expressions in the FCN model of Lambert et al. (2007)

$$X_{FCN} = A_C \cos(\sigma_{FCN} t) - A_S \sin(\sigma_{FCN} t)$$

$$Y_{FCN} = A_S \cos(\sigma_{FCN} t) + A_C \sin(\sigma_{FCN} t)$$

- partial derivative of the time delay w.r.t. the FCN frequency (contained in the motion of the CIP)

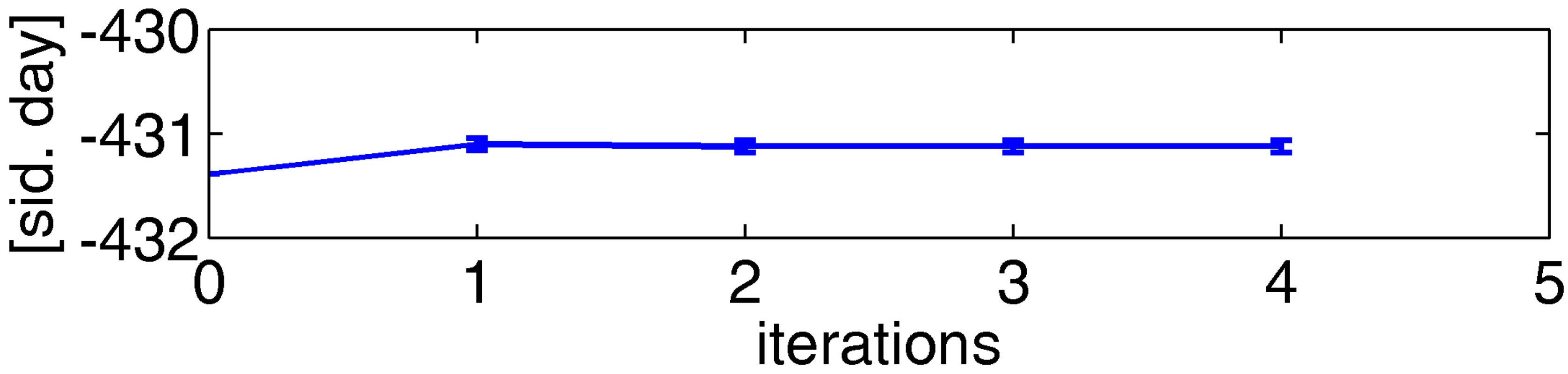
$$\frac{\partial \tau}{\partial \sigma_{FCN}} = k \cdot \frac{\partial dQ}{\partial \sigma_{FCN}} Q_{IAU} RW \cdot b$$

k – source vector
QRW – transformation matrix
b – baseline vector

FCN period as global parameter from the motion of the CIP (2)

- Solution 1
  - a priori value for FCN period: -431.39 sid. days (value in IERS2010)
  - a priori values for the FCN amplitudes: Lambert et al. (2007)

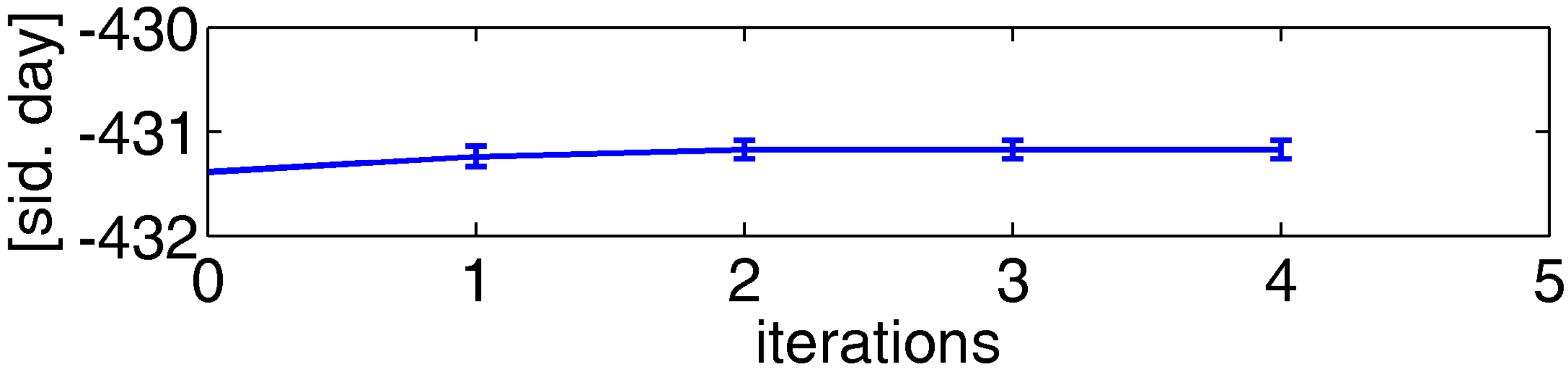
  - amplitudes were fixed to a priori values
  - estimated FCN period: **-431.12 ± 0.06 sidereal days**



FCN period as global parameter from the motion of the CIP (3)

- Solution 2
  - a priori value for FCN period: -431.39 sid. days (value in IERS2010)
  - a priori values for the FCN amplitudes: Lambert et al. (2007)

  - estimated global constant amplitude (sine and cosine part):
    Ac = 65 ± 1 µas, As = 35 ± 1 µas →
    A = 73 ± 1 µas, Φ = 28.03 ± 1.62 deg
  - estimated FCN period: **-431.17 ± 0.09 sidereal days**



FCN period as a global parameter from the motion of the CIP and from the solid Earth tides

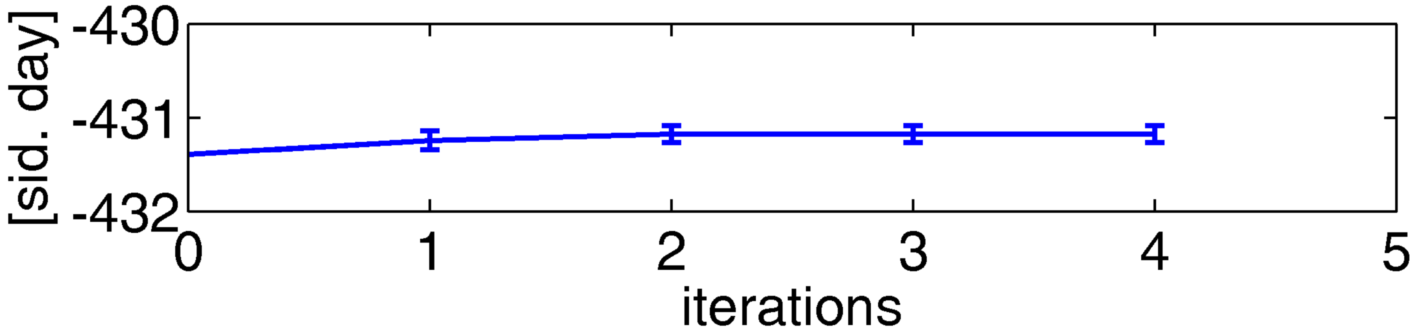
$$\frac{\partial \tau}{\partial \sigma_{FCN}} = \boxed{k \cdot \frac{\partial dQ}{\partial \sigma_{FCN}} Q_{IAU} RW \cdot b} + \boxed{k \cdot dQ \cdot Q_{IAU} RW \cdot \frac{\partial b}{\partial \sigma_{FCN}}}$$

Partial derivative of the matrix dQ, which contains the contribution of the FCN to the precession-nutation.

Partial derivative of the basline b, which contains the stations displacement caused by solid Earth tides.

a priori FCN period: -431.39 sidereal days (IERS2010)
a priori amplitude of the FCN: taken from Lambert et al. (2007)

Estimated FCN period: **-431.18 ± 0.10 sidereal days**



Summary I

- Software VieVS allows to estimate in a common global adjustment a consistent TRF, CRF and EOP (VLBI data 1984.0 – 2011.0).

- In addition we estimated complex Love and Shida numbers from the diurnal band.

  - Differences in the corrections due to the frequency dependence of the Love and Shida numbers for diurnal tides between our estimates and those computed from the model (IERS2010) do not exceed 0.3 mm.

- Spectral analyses of the CPO show broad peaks and also double peaks around the periods, where the FCN period is expected.

  - We estimated constant amplitudes in the global adjustment of the VLBI data, which correspond to the two periods obtained by the spectral analysis (P1 = -415 solar days and P2 = -451 solar days).

  - The model (superposition of two harmonic waves) shows a good agreement with the residuals to the IAU 2006/2000A precession-nutation model, but it does not have any clear geophysical meaning.

Summary II

- FCN period was estimated as a global parameter in an adjustment of VLBI data (1984.0 – 2011.0)
  - from solid Earth tides: **-431.23 ± 2.4 sidereal days**
  - from the motion of the CIP in GCRS:

Sol.	P_0 [sid. days]	A_{C0} [μas]	A_{S0} [μas]	P [sid. days]	A_C [μas]	A_S [μas]	A [μas]	Φ [deg]
1	-431.39	Lambert (2007)		-431.12 $\pm$ 0.06	-	-	-	-
2	-431.39	Lambert (2007)		-431.17 $\pm$ 0.09	65 $\pm$ 1	35 $\pm$ 1	73 $\pm$ 1	28.03 $\pm$ 1.62

  - from both phenomena simultaneously: **-431.18 ± 0.10 sidereal days**
  - Our results for the FCN period are only slightly different from the value adopted in the IERS Conventions 2010: -431.39 sidereal days.

Thank you for your attention!

Hana Spicakova and Sigrid Böhm work within
FWF-Project P23143 „Integrated VLBI“.