Federal Agency for
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IVS combination center at BKG

Robust outlier detection and weighting strategies

Background:

Using the Least Median Square (LMS) Method within the VLBI combination leads to a robust outlier strategy based on statistic parameters which is required for the variance component estimation (VCE). The idea of introducing the influence by the software and the responsible operator on the analyzed data is investigated in the Operator-Software Impact method (OSI).

Least Median Square (LMS)

The LMS is a very robust estimator with respect to outliers and is highly resistant to leverage points (theoretical breakdown point of 50%).

The objective function can be expressed as $\min \operatorname*{med}_{i} \mathbf{v}_i^2$

Where $\mathbf{v}$ is the vector of residuals. Outliers can be identified by comparing the robust standardized residuals to a given Threshold k :

$$\nabla_i = \begin{cases} false, & if \left|\mathbf{v}_i\right| \le k\sigma_{\mathrm{LMS}} \\ true, & else \end{cases}$$

with

$$\sigma_{\mathrm{LMS}} = 1.4826\left(1+\frac{5}{n-u}\right)\sqrt{\min \operatorname*{med}_{i} \mathbf{v}_i^2}$$

(n and u are the number of observations and unknowns, respectively [1]).

Variance component estimation

1. With respect to the observations (VCEO)

$$\sigma_i^2 = \frac{\mathbf{v}_i^T\mathbf{P}_i\mathbf{v}_i}{r_i} = \frac{\left(\mathbf{A}_i\mathbf{x}_{\mathrm{c}}-\mathbf{l}_i\right)^T\mathbf{P}_i\left(\mathbf{A}_i\mathbf{x}_{\mathrm{c}}-\mathbf{l}_i\right)}{n_i - \mathrm{trace}\left(\mathbf{N}_{\mathrm{c}}^{-1}\mathbf{A}_i^T\mathbf{P}_i\mathbf{A}_i\right)} = \frac{\mathbf{x}_{\mathrm{c}}^T\mathbf{N}_i\mathbf{x}_{\mathrm{c}} - 2\mathbf{x}_{\mathrm{c}}^T\mathbf{n}_i + \mathbf{l}_i^T\mathbf{P}_i\mathbf{l}_i}{n_i - \mathrm{trace}\left(\mathbf{N}_{\mathrm{c}}^{-1}\mathbf{N}_i\right)}$$

where $\mathbf{v}$ = vector of residuals, r = redundancy and n = number of observations. This method proposed by [2] can be applied, if the system of normal equations $\mathbf{n}_i = \mathbf{N}_i\mathbf{x}_i$, the weighted sum of reduced observations $\mathbf{l}_i^T\mathbf{P}_i\mathbf{l}_i$ and the number of observations n_i are given.

2. With pseudo observations (VCEP)

Each single solution $\mathbf{x}_i = \mathbf{l}_i$ is a realization of a stochastic process whereas $\mathbf{N}_i^{-1}$ is considered as covariance matrix of $\mathbf{l}_i$.

$$\mathbf{C}_{\mathrm{c}} = \mathrm{blockdiag}\left(\sigma_1^2\mathbf{N}_1^{-1} \quad \sigma_2^2\mathbf{N}_2^{-1} \quad \cdots \quad \sigma_n^2\mathbf{N}_n^{-1}\right)$$

Operator-Software Impact (OSI)

This method takes into account, that the combined solution has a too optimistic formal error relative to the individual solutions [3].

The basic idea is to split up the vector of observations: $\mathbf{l}_i = \mathbf{l}_{\mathrm{init}} + \boldsymbol{\delta}\mathbf{l}_i$

where $\mathbf{l}_{\mathrm{init}}$ = the initial observations (used by each AC) and

$\boldsymbol{\delta}\mathbf{l}_i$ = influence of the individual AC. The vectors $\mathbf{l}_{\mathrm{init}}$ and $\mathbf{l}_i$ are uncorrelated pair by pair. The different analysis strategies are modeled as stochastic effect:

$$\mathbf{C}_{\mathrm{OSI}} = \sigma_{\mathbf{l}_{\mathrm{init}}}^2 \begin{pmatrix} \mathbf{Q}_{\mathbf{l}_{\mathrm{init}}\mathbf{l}_{\mathrm{init}}} & \cdots & \mathbf{Q}_{\mathbf{l}_{\mathrm{init}}\mathbf{l}_{\mathrm{init}}} \\ \vdots & \ddots & \vdots \\ \mathbf{Q}_{\mathbf{l}_{\mathrm{init}}\mathbf{l}_{\mathrm{init}}} & \cdots & \mathbf{Q}_{\mathbf{l}_{\mathrm{init}}\mathbf{l}_{\mathrm{init}}} \end{pmatrix} + \sigma_{\boldsymbol{\delta}\mathbf{l}_1}^2 \begin{pmatrix} \mathbf{Q}_{\boldsymbol{\delta}\mathbf{l}_1\boldsymbol{\delta}\mathbf{l}_1} & \mathbf{0} \\ & \\ \mathbf{0} & \mathbf{0} \end{pmatrix} + \ldots + \sigma_{\boldsymbol{\delta}\mathbf{l}_n}^2 \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ & \\ \mathbf{0} & \mathbf{Q}_{\boldsymbol{\delta}\mathbf{l}_n\boldsymbol{\delta}\mathbf{l}_n} \end{pmatrix}$$

For the unknown matrix $\mathbf{Q}_{\boldsymbol{\delta}\mathbf{l}_i\boldsymbol{\delta}\mathbf{l}_i}$, two possible mathematic proofs have been proposed by [3]:

$$\mathbf{Q}_{\boldsymbol{\delta}\mathbf{l}_i\boldsymbol{\delta}\mathbf{l}_i} = \alpha_i^2\mathbf{I} \quad \text{and} \quad \mathbf{Q}_{\boldsymbol{\delta}\mathbf{l}_i\boldsymbol{\delta}\mathbf{l}_i} = \alpha_i^2\mathbf{Q}_{\mathbf{l}_{\mathrm{init}}\mathbf{l}_{\mathrm{init}}}$$

with the OSI-parameter $\alpha_i^2 > 0$.

Finally, α_i^2 serves as weighting factor for the covariance matrix $\mathbf{N}_c^{-1}$.

Literature

[1] Rousseeuw, P.J., Leroy, A.M.: Robust Regression and Outlier Detection. JohnWiley & Sons, New Jersey, ISBN: 978-0471488552, 2003.

[2] Vennebusch, M., Böckmann, S., Nothnagel, A.: The contribution of Very Long Baseline Interferometry to ITRF2005. J Geo, Vol 81, pp 553-564, doi:10.1007/s00190-006-0117-x, 2006.

[3] Kutterer, H. , Krügel, M., Tesmer, V.: Towards an improved assessment of the quality of terrestrial reference frames geodetic reference frames. In: Drewes H (ed) International Association of Geodesy Symposia, Vol 134. Springer, Berlin, pp 67–72. doi:10.1007/978-3-642-00860-3_10, 2009.

Further Information

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