WIDEBAND RFI MITIGATION

2017-06-08

Department of Electrical & Electronic Engineering,
Stellenbosch University

Jan-Willem W Steeb

Supervisor: Prof David B Davidson
Distinguished Professor and SKA SARCHI Chair, Department of E & E Engr, Stellenbosch University

Co-supervisor: Prof Stefan J Wijnholds
Extraordinary Associate Professor, Department of E & E Engr, Stellenbosch University and
System Researcher, Netherlands Institute for Radio Astronomy (ASTRON), The Netherlands

What is a Narrowband Signal?

R. T. Compton's Definition [2]:
"There is essentially no decorrelation between signals at opposite ends of the array."

M. Zatman's Definition [1]:
"If the bandwidth of a signal is such that the second eigenvalue of the signal's noise-free covariance matrix is larger than the noise level in the signal-plus-noise covariance matrix, then that signal may not be described as narrowband."

Video of Point Source as Bandwidth Increases

Eigenvalues versus Bandwidth

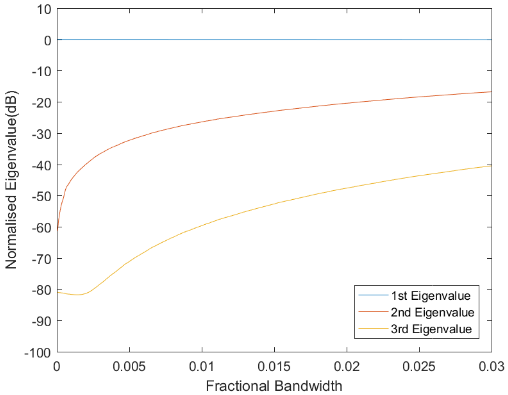


Figure: Normalised Eigenvalues as a function of fractional bandwidth using the layout of the LOFAR CS302 station.

Signal Assumptions

- Single RFI point source.
- RFI Source lies in the far field.
- RFI Source has approximately a flat frequency response for the entire channel bandwidth.
- System noise is Gaussian and independent and identically distributed for all antennas.
- The signal is wideband according to Zatman's definition (narrowly wideband).

Covariance Matrix

The covariance matrix for an array were there is one wideband RFI source and identically distributed noise is given by

$$\mathbf{R} = \frac{1}{\Delta\nu}\int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \sigma_r^2(\nu)\mathbf{a}(\nu)\mathbf{a}^H(\nu)d\nu + \sigma_n^2\mathbf{I}, \tag{1}$$

where the normalised array response vector is given by

$$\mathbf{a} = \frac{1}{\sqrt{N_e}}\begin{bmatrix} e^{-2\pi j\nu\tau_1} \\ \vdots \\ e^{-2\pi j\nu\tau_{N_e}} \end{bmatrix}. \tag{2}$$

Flat Frequency Response Covariance Matrix

If a flat frequency response is assumed, the integral for the covariance matrix is

$$\mathbf{R} = \frac{\sigma_r^2}{\Delta\nu} \int_{\nu_0-\Delta\nu/2}^{\nu_0+\Delta\nu/2} \mathbf{a}(\nu)\mathbf{a}^H(\nu)d\nu + \sigma_n^2\mathbf{I}. \tag{3}$$

Evaluating this integral yields

$$r_{ik} = \sigma_r^2 \,\mathrm{sinc}\left(\tau_{ik}\Delta\nu\right)\frac{e^{-2\pi j\nu_0\tau_{ik}}}{N_e}. \tag{4}$$

For Compton's definition $\mathrm{sinc}(\tau_{ik}\Delta\nu) \simeq 1$ (where τ_{ik} is the longest delay).

Zatman's Approximation

If the covariance matrix has an effective rank of two it can be approximated by the sum of two discrete uncorrelated signals

$$\mathbf{R}_r \approx \sigma_1^2 \mathbf{a}_1 \mathbf{a}_1^H + \sigma_2^2 \mathbf{a}_2 \mathbf{a}_2^H. \tag{5}$$

Zatman's model requires that the two signals have equal power

$$\sigma_1^2 = \sigma_2^2 = \sigma^2, \tag{6}$$

which is achieved when the discrete power sources are so arranged that the instantaneous frequency spectrum mean and variance correspond to those of the non-zero bandwidth signal. Consequently, the spacing from the centre frequency ν_0 is given by

$$\kappa = \frac{\Delta\nu}{2\sqrt{3}}. \tag{7}$$

Zatman's Approximation

The two source model is now changed to

$$\mathbf{R}_r = \sigma^2[\mathbf{a}(\nu_0 + \kappa)\mathbf{a}(\nu_0 + \kappa)^H + \mathbf{a}(\nu_0 - \kappa)\mathbf{a}(\nu_0 - \kappa)^H]. \quad (8)$$

The RFI covariance matrix can be decomposed using eigenvalue decomposition

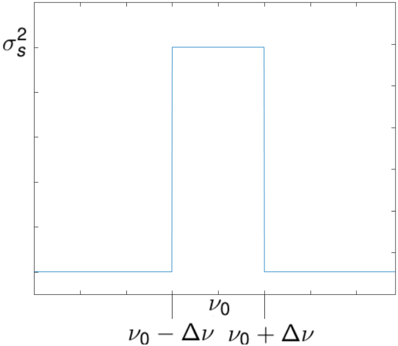
$$\mathbf{R}_r = \lambda_1 \mathbf{v}_1 \mathbf{v}_1^H + \lambda_2 \mathbf{v}_2 \mathbf{v}_2^H, \quad (9)$$

where $\lambda_1 > \lambda_2$. Using the properties of eigenvectors the second eigenvector can be reconstructed

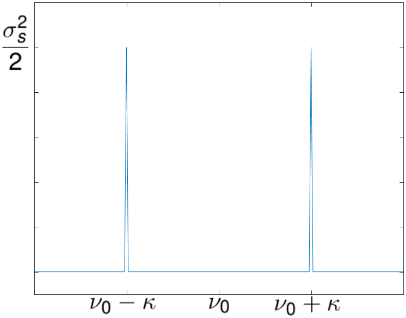
$$\mathbf{v}_2 = \beta_2 \left[\mathbf{a}_1 - \frac{\psi^*}{|\psi|} \mathbf{a}_2 \right], \quad (10)$$

where $\psi = \frac{\mathbf{a}_1 \mathbf{a}_2^H}{N_e}$ and $\beta_2 = \sqrt{\frac{1}{2(1-|\psi|)}}$.

Model Summary



(a) Periodogram of flat frequency response model.



(b) Periodogram of Zatman's two source model.

RFI Mitigation: Orthogonal Projection

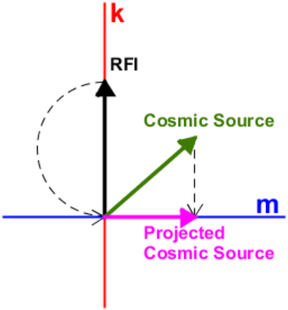


Figure: Illustration of the orthogonal projection applied to a 2-dimentional data vector space.

If the column vectors of $\mathbf{A}_r$ are linearly independent, they form a basis for a vector space V_r. Therefore, an orthogonal projector can be constructed [3, p. 430]

$$\mathbf{P} = \mathbf{I} - \mathbf{A}_r(\mathbf{A}_r^H\mathbf{A}_r)^{-1}\mathbf{A}_r^H, \qquad (11)$$

such that $\mathbf{PA}_r = \mathbf{0}$. Applying the projector to the data model

$$\begin{aligned}\mathbf{PRP} &= \mathbf{PR}_c\mathbf{P} + \mathbf{PA}_r\mathbf{B}_r\mathbf{A}_r\mathbf{P} + \mathbf{PR}_n\mathbf{P} \\ &= \mathbf{PR}_c\mathbf{P} + \mathbf{PR}_n\mathbf{P}\end{aligned} \qquad (12)$$

Flat Frequency Response Based Algorithm

Preprocessing:

- Use singular value decomposition $\mathrm{eigen}(\mathbf{R}) = \mathbf{VsV}$ and let $\mathbf{v}_1 = \mathbf{V}(:, 1)$.
- Use an algorithm such as MUSIC or ESPRIT to determine the direction of arrival (l, m).

Flat frequency response model based approach

- Calculate $\widetilde{\mathbf{R}}_r$ using $r_{ik} = \sigma_r^2 \,\mathrm{sinc}\left(\tau_{ik}\Delta\nu\right)\frac{e^{-2\pi j \nu_0 \tau_{ik}}}{N_e}$.
- Apply eigenvalue decomposition $\mathrm{eigen}(\widetilde{\mathbf{R}}_r) = \widetilde{\mathbf{V}}\widetilde{\mathbf{s}}\widetilde{\mathbf{V}}$ and set $\widetilde{\mathbf{v}}_2 = \widetilde{\mathbf{V}}(:, 2)$.
- Construct an orthogonal projector

$$\mathbf{P} = \mathbf{I} - [\mathbf{v}_1, \widetilde{\mathbf{v}}_2]([\mathbf{v}_1, \widetilde{\mathbf{v}}_2]^H[\mathbf{v}_1, \widetilde{\mathbf{v}}_2])^{-1}[\mathbf{v}_1, \widetilde{\mathbf{v}}_2]^H \tag{13}$$

Zatman's Approximation Based Algorithm

Preform same prepossessing as for flat frequency response based algorithm.

Zatman's approximation based approach

- Construct

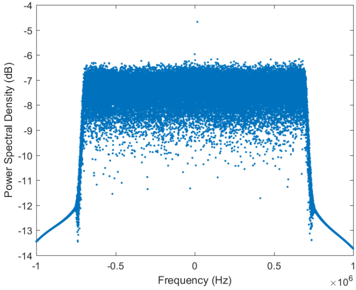
$$\widetilde{\mathbf{v}}_2 = \beta_2 \left[\mathbf{a}_1 - \frac{\psi^*}{|\psi|} \mathbf{a}_2 \right] \tag{14}$$

where $\psi = \frac{\mathbf{a}_1 \mathbf{a}_2^H}{N_e}$ and $\beta_2 = \sqrt{\frac{1}{2(1-|\psi|)}}$.

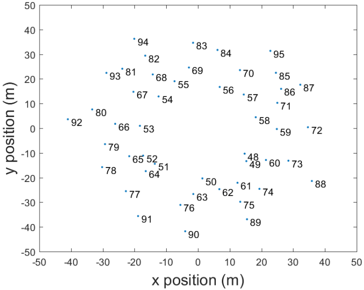
- Construct an orthogonal projector

$$\mathbf{P} = \mathbf{I} - [\mathbf{v}_1, \widetilde{\mathbf{v}}_2]([\mathbf{v}_1, \widetilde{\mathbf{v}}_2]^H [\mathbf{v}_1, \widetilde{\mathbf{v}}_2])^{-1} [\mathbf{v}_1, \widetilde{\mathbf{v}}_2]^H. \tag{15}$$

Simulation Setup

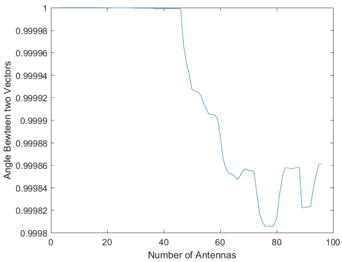


(a) Periodogram of DAB signal.

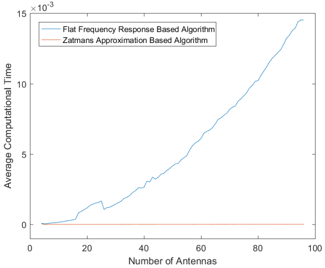


(b) LOFAR LBA Station CS302 layout.

Simulation Results

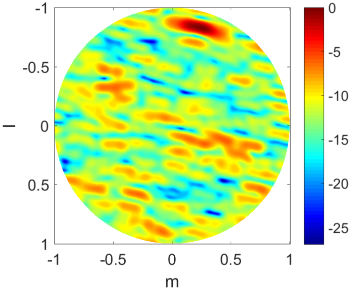


(a) Angle between flat frequency response model vector and Zatman's approximation model vector.

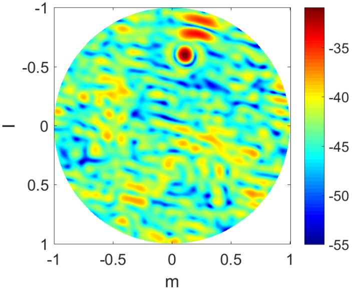


(b) Computation time averaged for 1000 runs as a function of the number of antennas.

Simulation Results

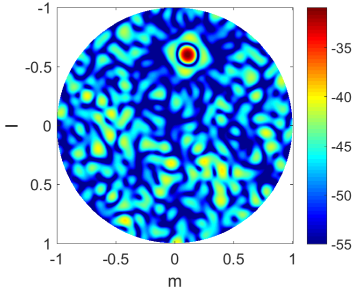


(a) Full skymap with wideband RFI source visible in top right corner in dB (the RFI source is the 0 dB point).

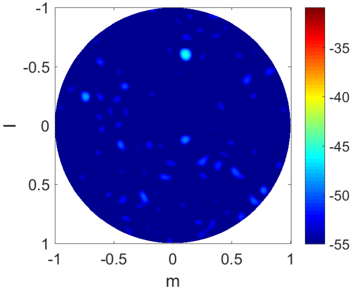


(b) Full skymap with RFI source removed using orthogonal projection with bias correction in dB.

Simulation Results

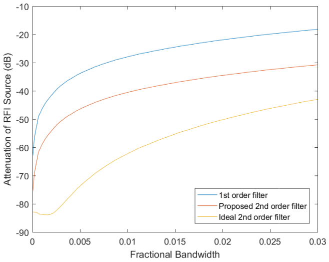


(a) Full skymap with RFI source removed using the adapted orthogonal projection with bias correction in dB. The secondary RFI sources are removed and only the cosmic source is present.

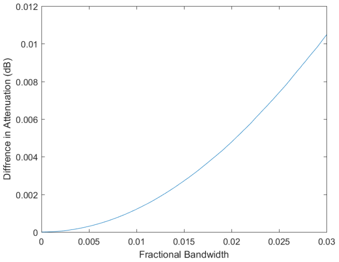


(b) Difference between skymaps obtained from ideal 2nd order filter and proposed 2nd order filter.

Simulation Results



(a) Comparison of the attenuation of an RFI signal as a function of fractional bandwidth for a 1st order ortogonal projection filter, 2nd order orthogonal projection filter where the 2nd eigenvector is constructed from the first and 2nd order orthogonal projection filter.



(b) The difference in attenuation between the flat frequecny response method and Zatman's approximation based method.

Bibliography I

- M. Zatman, “How narrow is narrowband?,” *IEE Proceedings-Radar, Sonar and Navigation*, **145**, 2, April 1998, pp. 85–91, doi:10.1049/ip-rsn:19981670.
- R. T. Compton, “Adaptive Antennas: Concepts and Performance,” 1988, Prentice Hall.
- C. D. Meyer, “Matrix Analysis and Applied Linear Algebra” 2001, SIAM: Society for Industrial and Applied Mathematics.