Proceedings of the 7th European VLBI Network Symposium
Bachiller, R. Colomer, F., Desmurs, J.F., de Vicente, P. (eds.)
October 12th-15th 2004, Toledo, Spain

What is the primary beam response of an interferometer with unequal elements?

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Abstract. The EVN stations encompass elements with a range of diameters, even including an interferometer (the Westerbork Telescope, with up to 14 elements used together as a tied array). In combination, the various station pairs will each produce their own primary beam envelopes, with which the interferometer pattern is modulated. People sometimes forget that in the case of unequal elements, this combined primary beam envelope is different from the beam of each element separately. The reason for this is reviewed, the results for a number of station pairs are summarized, and some of the practical consequences are discussed. The increased interest in wide-field applications, as illustrated by several recent results, underlines the need for a proper determination of the interferometer beam envelope.

1. Introduction

This short note is motivated by comments I have occasionally heard (or read in proposals or similar unpublished documents), in the hope that it will correct a misconception some people seem to have. The argument goes something like this: We want an in-beam phase reference. Hmmm, our largest element will be Effelsberg, it has a beamsize (FWHM) of 7.4′ arc at 18 cm. We need to find a phase reference within 3.7′ of our target source. Reasoning like this reveals a misconception which, though understandable in some respects, is nonetheless fundamentally wrong. In the following paragraphs I will try to explain why.

Before launching into a more detailed discussion, I would first like to state some facts about radio telescopes which most readers are probably familiar with. There are two parameters which characterize the sensitivity of a telescope: the effective collecting area, and the system temperature. The effective area for a reflector is, of course, the physical area multiplied by the efficiency (which takes into account the surface accuracy, leakage if the surface is not solid, and the feed illumination pattern). For a point source of strength S, the effective area A_e comes into the well-known formula, $S = 2kT/A$, whereby the ratio, S/T, which tells us the source flux density required to raise the receiver temperature by 1 K (usually expressed in Jy/K), is seen to be determined by A_e. We can then derive the system noise in Jy (SEFD) from the system temperature. The receiver bandwidth can then be used to calculate the noise level after a certain integration time. (All of the above can be found in standard references on antennas and radio astronomy, such as Kraus (1986).)

There are two other simple relationships for a two-element interferometer which we might want to remember. Suppose that the elements, labelled 1 and 2, have collecting areas A_1 and A_2, and system temperatures T_1 and T_2. Then the interferometer response (ignoring the individual element responses) for each of these quantities will simply be the geometrical mean of element 1 and 2. For the interferometer collecting area we have,

$$A_{int} = (A_1 A_2)^{1/2}$$

And for the combined system temperature,

$$T_{int} = (T_1 T_2)^{1/2}$$

It should become clear later on why this is so.

2. Overview of the EVN elements

The EVN consists of ten regular stations, often augmented with a handful of other ones. The regular station elements range in diameter from 100 m to 25 m. An exceptional case is the Westerbork Telescope array, which consists of fourteen 25 m dishes on a 2.7 km east-west baseline. When phased-up and added together, it has the equivalent of a roughly 90 m dish. Fewer Westerbork dishes can also be added (if just twelve are included, the array length is decreased to 1400 m and the effective diameter is about 85 m). Westerbork can also be used as a 25 m single dish (Wb(1)). All of the stations which participate in the EVN are listed in Table 1.

The Arecibo primary is 305 m (1000 ft) in diameter, but for normal observations about 188 m of this is illuminated (Heiles et al. 2001; the measured FWHM suggests a larger value). Wz mainly participates in geodetic observations (and only operates at S/X-band). The other small elements are mainly for high frequency observing.

3. Response of a single dish

Let us begin by considering the response of a single dish, as that will provide us with some fundamentals before we turn to the case of an interferometer. The antenna response, A, can be found by taking the Fourier transformation (FT) of the autocorrelation of the aperture illumination, v. For simplicity I will take a one-dimensional case, with u the coordinate in the aperture plane (u expressed in wavelengths), and x the coordinate

Table 1. Diameters of EVN elements.

Diameter	Station(s)
200 m	Ar
100 m	Eb
90 m	Wb
76 m	Jb1
70 m	Rb70
32 m	Cm, Mc, Nt, Tr, Rb34
25 m/85 ft	Hh, Jb2, On – 85, Sh, Ur, Wb(1)
20 m	On – 60, Wz
14 m	Mh, Yb

in the sky or antenna beam frame (in radians). The autocorrelation of $v(u)$ has the usual definition, $a(u) = \int v(l)\, v(u + l)\, dl$, or in simplified notation, $a = v * v$. (I will assume that the telescope response is symmetrical, and ignore the difference between correlation and convolution.)

The Fourier transformations of a and v will, as is the usual convention, be denoted by A and V, respectively. The FT of a has the usual definition: $A(x) = \int a(u) \exp(-2\pi i\, ux)\, du$. In shorthand notation, $a(u) \rightarrow A(x)$. We want to determine $A(x)$, and we have the following relationship:

$$v * v = a \rightarrow A$$

We can use the convolution theorem (the FT of the convolution of two functions is the product of their individual convolutions), to get:

$$A = V \times V$$

So, $A(x) = V^2(x)$, where $A(x)$ is the power polar diagram, while $V(x)$ is the voltage polar diagram.

4. Primary beam envelope of an interferometer

What happens in the case of an interferometer? Then for our $v(u)$ we have to take $v_1(u)$ and $v_2(u)$ for our two elements 1 and 2, and separate them by some distance, Δu (= interferometer baseline). This gives us a new aperture illumination, $v'(u) = v_1(u) + v_2(u + \Delta u)$, so the autocorrelation we want is, $(v_1(u)+v_2(u+\Delta u))*(v_1(u)+v_2(u+\Delta u))$. However, we are only interested in the beam response envelope, so we will ignore the autocorrelations ($v_1 * v_1$, etc.), and assume that our elements 1 and 2 coincide ($\Delta u = 0$), to get the cross-correlation of the element illumination patterns, $a_{12} = v_1 * v_2$. It is the FT of this that we want:

$$a_{12} \rightarrow A_{12}$$

and as in the case of the single dish, we get

$$A_{12} = V_1 \times V_2$$

The combined beam envelope response is simply the voltage polar diagram of one multiplied by that of the other.

The consequences of this can be most simply seen by considering a limiting case. Suppose that the elements of our interferometer, with diameters D_1 and D_2, are very unequal with $D_1 >> D_2$. Then their beam sizes (θ) will also be unequal, $\theta_1 << \theta_2$. Now, θ_2 large means $V_2(x) \simeq \text{const} = V_0$, which gives,

$$A_{12}(x) \simeq V_1(x) \times V_0$$

and the width of the combined interferometer beam will be determined by the *voltage* pattern, V_1. For a (nearly) gaussian-shaped beam, the FWHM response will be about 40% larger (greater by $\sqrt{2}$ for a true gaussian), and the beam area will of course be doubled. In reality, the illumination pattern (even if gaussian) will be truncated at the edge of the aperture, leading to sidelobes and nulls in the antenna pattern. The nulls will, naturally, have the same location in the voltage beam, so that will not be changed in the interferometer response. The relative strength of the sidelobes will be increased.

Table 2. EVN element and interferometer beam properties at 18 cm.

$\mathrm{Sta}_1 * \mathrm{Sta}_2$	θ_1	θ_2	θ_{12}	Ω_1/Ω_{12}
Ar * Eb	2.9′ arc	7.4′ arc	3.8′ arc	1.74
Ar * Jb1	2.9′	10′	3.9′	1.84
Ar * 32 m	2.9′	23.2′	4.1′	1.97
Ar * 25 m	2.9′	30′	4.1′	1.98
Eb * Jb1	7.4′	10′	8.4′	1.3
Eb * 32 m	7.4′	23.2′	10′	1.83
Eb * 25 m	7.4′	30′	10.2′	1.9
Jb1 * 25 m	10′	30′	13.4′	1.8
32 m * 25 m	23.2′	30′	26.0′	1.25

5. Primary beam envelope of EVN interferometers

As we saw in Section 2, the EVN station elements differ considerably in size (Table 1). Here I would like to consider the consequences of these differences on the combined beam envelope for some common baseline configurations. The calculations have been done for a wavelength of 18 cm, as this is widely used in the EVN and encompasses all stations. For Arecibo, the beamwidth (FWHM) measured by Heiles et al. (2001) at 1666 MHz has been used. For Effelsberg, the value measured by Reich et al. (1978) at 2700 MHz has been scaled to 1666 MHz. For the other elements, the Effelsberg value has been scaled by the dish diameter. Table 2, most of which is self-explanatory, gives the results for most baseline combinations of elements ranging from Arecibo and Effelsberg, to the 32 and 25 m dishes. The fourth column, headed θ_{12}, is the combined interferometer beam envelope for the elements in column 1. The fifth column, Ω_1/Ω_{12}, is the factor by which the FWHM interferometer beam area is greater than the single-dish beam area of the larger element.

The increase in beam area for the interferometer pair over that of the larger element is naturally greater when the elements have very different diameters. The Arecibo and Effelsberg combinations with the smaller elements illustrate this quite clearly. There is, of course, no sharp "edge" to the primary beam, so one has considerable discretion in deciding where to draw the line, but even taking it at $2/\pi$ or $1/e$ or whatever one's favorite cutoff might be, within reason, similar ratios will hold. The

Westerbork array has not been considered here, as it is a rather exceptional case. But for it too, the width of the fan beam at 18 cm will increase from about 20″ arc to 28″ arc. However a full treatment is beyond the scope of this short note.

Users of arrays with equal elements, like the VLBA, do not have to concern themselves with these matters, except in the case of global experiments (but then, that's no longer the VLBA as such, nor are the elements equal).

6. So what?

Most VLBI users are only concerned with emission within the inner $< 1''$ arc of their pointing direction. For them, the points raised above may be of no more than academic interest. However, for those wanting to pursue the structure of numerous sources in a larger region of the sky, it is worth considering the correct (combined) beam size. The area covered (and hence the number of sources) can be nearly double that of the larger element used as a single dish. This having been said, most of the beam sizes in Table 2 extend well beyond the distance where bandwidth decorrelation and integration time smearing will normally limit effective imaging. The most likely consequence of the beamsizes lies in the realm of using in-beam phase reference sources. Doubling the beam area means doubling the chance of finding a suitable reference.

There may, in the future, be better prospects for imaging over a larger portion of the primary beam envelope. With correlators and storage devices able to handle larger data volumes, the possibilities for producing many frequency channels with short integration times will also increase. From the observational side, there is increasing pressure for wider fields of view. This is aptly illustrated by recent EVN observations of the Hubble Deep Field (Garrett et al. 2001). Similarly, the combination of EVN and MERLIN also tends to move one into the realm of wide(r)-field mapping.

7. Conclusions

The results presented here on the primary beam of an interferometer with unequal elements should be familiar to anyone working in the field of radio interferometry. Unfortunately, my experience has been that these facts are often overlooked. In stating the obvious, I hope that I have not bored too many readers.

Acknowledgements. I thank various colleagues for discussions held on this and realted topics over the past few years. ASTRON is funded by the Netherlands Organization for Scientific Research (NWO).

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