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Multi-frequency imaging in VLBI

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Abstract.
The new technique, *multi-frequency imaging* (MFI) is developed. In VLBI, Multi-Frequency Imaging (MFI) consists of multi-frequency synthesis (MFS) and multi-frequency analysis (MFA) of the VLBI data obtained from observations on various frequencies. A set of linear deconvolution MFI algorithms is described. The algorithms make it possible to obtain high quality images interpolated on any given frequency inside any given bandwidth, and to derive reliable estimates of spectral indexes for radio sources with continuum spectrum. Thus MFI approach makes it is possible not only to improve the quality and fidelity of the images and also essentially to derive the morphology of the observed radio sources.

1. Introduction

The new technique, *multi-frequency imaging* (MFI), is a powerful tool not only for *multi-frequency synthesis* (MFS) (Conway et al. (1990)), but also for *multi-frequency analysis* (MFA) of the VLBI data obtained from observations on various frequencies. This tool allows us to obtain both *high quality images interpolated on a given reference frequency* inside of a given bandwidth, as well as reliable estimates of *spectral indexes* for radio sources with continuum spectrum. This approach is very important not only for future Space VLBI missions like Radioastron, but also for ground-based VLBI arrays like the EVLA and VLBA. MFI approach makes it is possible not only to improve the quality and fidelity of the images and also essentially to derive the morphology of the observed radio sources. At the same time, this approach shows that MFI will provide the highest angular resolution possible for a given Space VLBI mission.

2. Statement of the problem

Let us consider a linear model for intensity $I_{kpq} = I(x_p, y_q, \nu_k)$ of the radio source in a point (x_p, y_q) on the observational frequency ν_k:

$$I_{kpq} \approx (I_0)_{pq} + (I_1)_{pq}\beta_k + \ldots + (I_{N-1})_{pq} \cdot (\beta_k)^{N-1},$$

$$\beta_k = \frac{\nu_k}{\nu_0} - 1, \quad k = 1,\ 2,\ \ldots\ ,\ K,$$

where ν_0 is reference frequency corresponding to the intensity $(I_0)_{pq}$.

If the intensity I_{kpq} in the point (x_p, y_q) can be approximated by power law as

$$I_{kpq} = (I_0)_{pq} \cdot \left(\frac{\nu_k}{\nu_0}\right)^{\alpha_{pq}},$$

then we can present it as

$$I_{kpq} = (I_0)_{pq}\, e^{\xi_k \alpha_{pq}} \approx (I_0)_{pq} \cdot \left(1 + \xi_k \alpha_{pq}\right)$$

where $\xi_k = \ln\ (1 + \beta_k) \approx \beta_k$,

and thus the spectral indexes $\alpha_{pq} = \alpha\left(x_p, y_q\right)$ can be obtained as

$$(I_1)_{pq} = \alpha_{pq} \cdot (I_0)_{pq}$$

Let us consider a target function

$$\rho = \sum_{k=1}^{K} \sum_{n=0}^{M-1} \sum_{m=0}^{M-1} w_{knm} \cdot \left|V_{knm} - \hat{V}_{knm}\right|^2,$$

where, $w_{knm} = w(u_n, v_m, \nu_k) \geq 0$ are weights, V_{knm}, $\hat{V}_{knm}$ is a measured and a model visibility function respectively,

$$\hat{V}_{knm} = A_k \cdot \sum_{p,q=o}^{M-1} \left[\sum_{l=o}^{N-1} \left(\widehat{I_l}\right)_{pq} \cdot \beta_k^l\right] \cdot \exp\left\{-2\pi i \cdot \left(u_n x_p + v_m y_q\right)\right\},$$

where, A_k is a gain coefficient for k-th antenna,

$$\left(\widehat{I_l}\right)_{pq} = \Delta^2 \varphi_{pq} \cdot (I_l)_{pq} \left(1 - x_p^2 - y_q^2\right)^{-0.5},$$

φ_{pq} is a normalized beam, Δ is a grid step.

The problem of the optimization can be presented as a solution of the following system of linear equations:

$$(D_0)_{pq} = 0,\ \ldots,\ (D_{N-1})_{pq} = 0$$

for a vector of intensity $\left(\widehat{\mathbf{I}}\right)_{rt} = \left(\left(\widehat{\mathbf{I}}_0\right)_{rt}, \left(\widehat{\mathbf{I}}_1\right)_{rt}, \ \ldots,\ \left(\widehat{\mathbf{I}}_{N-1}\right)_{rt}\right)^T$,

where the m-th residual map $(D_m)_{pq}$ can be defined as:

$$(D_m)_{pq} = \sum_{k=1}^{K} \beta_k^m \times$$

$$\times \left\{D_{kpq} - \sum_{i=0}^{M-1} \sum_{l=0}^{M-1} B_{k,p-i,q-l} \cdot \sum_{n=0}^{N-1} \left(\widehat{\mathbf{I}}_n\right)_{il} \cdot \beta_k^n\right\}, \tag{1}$$

$$m = 0,\ 1,\ \ldots,\ N-1, \tag{2}$$

where,

$$D_{kpq} = \sum_{n,m=o}^{M-1} w_{knm} \cdot V_{knm} \cdot \exp\left\{2\pi i \left(u_n x_p + v_m y_q\right)\right\}$$

is a k-th "dirty" map at the point $\left(x_p, y_q\right)$,

$$B_{k,p-i,q-l} = \sum_{n,m=o}^{M-1} w_{knm} \times$$

$$\times \exp\left\{2\pi i \left[u_n \left(x_p - x_i\right) + v_m \left(y_q - y_l\right)\right]\right\}$$

is a k-th "dirty" beam at the point $\left(x_p - x_i, y_q - y_l\right)$.

3. Solution of the problem

The solution of the problem can be presented as an iterative procedure for a vector $\left(\widehat{\mathbf{I}}\right)_{pq}$:

$$\left(\widehat{\mathbf{I}}\right)_{pq}^{(s)} = \left(\widehat{\mathbf{I}}\right)_{pq}^{(s-1)} + \gamma \mathbf{E}^{-1} \cdot (\mathbf{D})_{pq}^{(s-1)},$$

and the residual maps $(\mathbf{D})_{rt} = \{(D_0)_{rt}, (D_1)_{rt}, ..., (D_{N-1})_{rt}\}^T$:

$$(\mathbf{D})_{rt}^{(s)} = (\mathbf{D})_{rt}^{(s-1)} - \widehat{\mathbf{B}}_{r-p,t-q} \cdot \left[\left(\widehat{\mathbf{I}}\right)_{pq}^{(s)} - \left(\widehat{\mathbf{I}}\right)_{pq}^{(s-1)}\right].$$

Here $\mathbf{E} = \left(E_{ij}\right)$ is a positive defined matrix of maximum values of weighted "dirty" beams, $E_{ij} = \left(\widehat{B}_{i+j}\right)_{0,0}, i, j = 0, ..., N-1$; γ is a loop gain. The process of the iteration can be completed if $\varepsilon^{(s-1)} < \varepsilon$, where ε is a given accuracy. Otherwise it is necessary to suppose $s = s + 1$ and to calculate the next $\varepsilon^{(s)}$.Conditions of the convergence of the algorithm above is $0 < \gamma < 2$, $1 \leq N \leq K$.

The developed algorithm is nothing other than the *multi-frequency linear deconvolution (multi-frequency CLEAN)*, itself. This procedure described in more detail by Likhachev et al. (2003). Notice that the developed algorithm allows to synthesize and analyze of high-quality VLBI images directly from the visibility data measured observed on a few frequencies, without analyses of the images of the source itself. In case of multi-frequency linear deconvolution, it is possible to synthesize an image of a radio source at any intermediate frequency *inside* any given frequency band. Thus, *spectral interpolation* of the image is feasible. This part of the algorithm is carry out the *synthesis* of the image itself. However, the algorithm also makes it possible to obtain an estimate of the *spectral index* for a given radio source, i.e., it implements the *analysis* of the image. It is clear that multi-frequency imaging (MFI) will provide the highest angular resolution possible for any VLBI project due to its improved (u, v)-coverage.

4. Implementation of the linear deconvolution algorithm

The algorithm described above was implemented in the software, ***Astro Space Locator (ASL) for Windows*** (`http://platon.asc.rssi.ru/dpd/asl/asl.html`).
It was developed by the Laboratory for Mathematical Methods of the Astro Space Center (Likhachev et al. (2003)).

References

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