What is the primary beam response of an interferometer with unequal elements?

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Rationale: comments such as –

- I want an in-beam reference source…

  Hmm…

  The largest element in the array has a FWHM beam of 9’ arc…

  I guess I should look for a source within 4.5’ arc of my target…
- Anything wrong with this? Uh, yes…

We all know the FWHM of a radio telescope:

- For a 25 m dish, ~30′ arc at 18 cm; ~10′ arc at 6 cm;
- For a 100 m dish, ~7.5′ arc at 18 cm; ~2.5′ arc at 6 cm;
- …etc.
- An interferometer then? For the VLBA it's simple: all elements are 25 m.

What’s the EVN situation?

Diameter	Station(s)
200 m (illuminated)	Ar
100 m	Eb
(14 x 25 m)=‘90 m’	Wb
76 m	Jb1
70 m	Rb70
32 m	Cm,Mc,Nt,Tr,Rb34
25 m/85 ft	Hh,Jb2,On85,Ur,Sh
20 m	On60, Wz

What happens with unequal elements, as in the EVN?

- Might guess that the beam will be some average of the two elements…
- Everyone should know that for elements with area A_1, A_2 and T_{sys} of T_1, T_2, the interferometer area and T_{sys} will be:

  $A_{12}=(A_1A_2)^{1/2}$ & $T_{12}=(T_1T_2)^{1/2}$

Consider the beamshape of a single dish

- The antenna response, *A*, is the FT of the autocorrelation of the aperture illumination, v: $v * v \Rightarrow A$

  ($\Rightarrow \equiv$ FT; $* \equiv$ correlation/convolution)
- Hence, $v \Rightarrow V$ & $a \Rightarrow A$
- Then, by the convolution theorem,

  $v * v \Rightarrow V \times V = A$

  (V is the antenna voltage pattern.)

What is the combined beamshape of an interferometer?

- The antenna response, A_{12}, is the FT of the cross-correlation of the 2 aperture illuminations: $v_1 * v_2 \Rightarrow A_{12}$
- Then, by the convolution theorem,

  $$v_1 * v_2 \Rightarrow V_1 \times V_2 = A_{12}$$
- For v_1 » v_2, V_2 ~ const, so $A_{12} \sim V_1$
- This results in a broader main beam lobe

EVN element and interferometer beam properties at 18 cm

$St_1 * St_2$	θ_1	θ_2	θ_{12}	$\Omega_{12}/\ \Omega_1$
Ar∗Eb	2.9′	7.4′	3.8′	1.74
Ar∗Jb1	2.9′	10′	3.9′	1.84
Ar∗Sh	2.9′	30′	4.1′	1.98
Eb∗Jb1	7.4′	10′	8.4′	1.3
Eb∗Sh	7.4′	30′	10.2′	1.9
Jb1∗Sh	10′	30′	13.4′	1.8
Mc∗Sh	23′	30′	26.0′	1.25

Interesting (?), but so what?

- If you’re only concern is what goes on within <1” arc, you’re not affected.
- If you want an in-beam reference source, then you might be interested.
- For wide-field mapping, this should certainly interest you!
- (And in any event, it’s fundamental.)

Postlude: what is the effect?

- Sources off axis will be attenuated in the sky plane (in the u,v–plane, the visibility will be convolved by the FT of the interferometer primary beam).
- This can be different for different baselines.
- Will affect map, but also self-cal.