



# Coordinate Based Bundle Adjustment

## Advanced Network Adjustment Model for Polar Measurement Systems

#### Abstract

To fulfil the GGOS requirements on local-ties at colocation stations, measurements with high precision instruments and data analysis with rigorous uncertainty propagation are necessary. To evaluate the results of e.g. high performance total stations or laser trackers, the accuracy-limiting parameters of the measurement process have to be quantified and projected onto an uncertainty model. Using the generally and interdisciplinarily accepted *Guide to the Expres*sion of Uncertainty in Measurement (GUM) a transparent and traceable stochastic model can be derived. A Cartesian coordinate based bundle adjustment is suggested to integrate the local measurements into a global context, avoiding gravitational influences. The included comprehensive uncertainty model is based on a specific geometric model of a polar measurement system and takes instrument specific and target dependent error parameters into account.

the level of Cartesian coordinates, the well known conversion has to be applied.

(1)

 $\mathbf{p}_{i,j}(\Theta, \Phi, d) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{i,j} = \begin{pmatrix} d\sin\Phi\cos\Theta \\ d\sin\Phi\sin\Theta \\ d\cos\Phi \end{pmatrix}_{i,j}$ 

#### **Deriving Uncertainties of Observations**

The use of Equation 1 assumes a perfect instrument which means, that:

- the azimuth and elevation axes are orthogonal to each other and have an intersection point,
- the distance measurement unit is centred w.r.t. this intersection point,
- the target beam is coaxial and orthogonal to the elevation axis,
- the normal vector of the angle encoder is aligned

Table 1: Calibration Parameters of a Polar Measurement Instrument.

Туре	Description		
λ	Displacement offset		
μ	Scaling parameter		
ex	Axis offset		
by	Y-laser offset		
bz	Z-laser offset		
<u>a</u>	Trunnion axis error		
γ	Horizontal collimation erro	or	
<b>a</b> (A,1)	Azimuth-encoder error,	1 <sup>th</sup> Fourier coefficient	
<b>b</b> (A,1)		1 <sup>th</sup> Fourier coefficient	
<b>a</b> (A,2)		2 <sup>nd</sup> Fourier coefficient	
<b>b</b> (A,2)		2 <sup>nd</sup> Fourier coefficient	
<b>a</b> (E,0)	Vertical collimation error		
<b>a</b> (E,1)	Elevation-encoder error,	1 <sup>th</sup> Fourier coefficient	
<b>b</b> (E,1)		1 <sup>th</sup> Fourier coefficient	
<b>a</b> (E,2)		2 <sup>nd</sup> Fourier coefficient	
<b>b</b> (E,2)		2 <sup>nd</sup> Fourier coefficient	

#### **Developed Prototype Software**

At the *Frankfurt Laboratory for Industrial Metrology* a prototype software has been developed to integrate local measurements into a global context (Lösler & Eschelbach 2012). Based on the compensation model of Hughes et al. (2011) an extended stochastic model with 25 parameters is implemented (Fig. 3).

#### **Concept of Rigorous Bundle Adjustment**

The network adjustment is the first and most important processing and conditioning step. As a result, the network adjustment provides the spatial coordinates and their corresponding uncertainties as full variance-covariance matrix. Depending on the extent of the local network and the accuracy requirements, the influence of the curvature of the earth cannot be neglected.

In metrology, coordinate based algorithms are developed, because most of the instruments are unrela-



and centred to the rotation axis, and

• the angle encoder is free of graduation errors. For manufacturing reasons, deviations from the idealized model can appear.



Fig. 2: Irregularities of a Polar Measurement Instrument (Hughes et al., 2011).

Hughes et al. (2011) formulated a 16 parameter comprehensive compensation model for a laser tracker and suggested a network configuration for deriving these calibration parameters (see Fig. 2). Whereas the calibration parameters are used to derive corrected Cartesian coordinates, the uncertainties of the calibration parameters and their related distributions have to be introduced to the stochastic model of the least-squares adjustment. The resulting correlations are also taken into account. Beside the propagation of uncertainty, Monte-Carlo simulation based techniques are proposed by the *Guide to the Expression of Uncertainty in Measurement* (GUM).

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Subset (3002)	instrument dependent		Distribution		
Subset (3004)	X-Value avo [m]	0Parameter 0.0002	Distribution	<u> </u>	
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Subset (3000)		0.0002	Uniform		
Subset (4001)		0.0002	Normal		
- Subset (4001)	Addition constant a wirel	0.00001	Normal		
- Subset (4003)	Addition constant oadd [m]	0.0006	Triangular		
- D Subset (4004)	Azimuth angle encoder daA,1 [gon]	0.0001	Uniform		
- D Subset (4007)	Azimuth angle encoder daA,2 [gon]	0.0001			
- Subset (4008)	Azimuth angle encoder obA,1 [gon]	0.0001	Normal		
- D Subset (3010)	Azimuth angle encoder σbA,2 [gon]	0.0001	Normal		
- Subset (3020)	Zenith angle offset oae,o [gon]	0.0003	Normal		
- Subset (3021)	Zenith angle encoder gae,1 [gon]	0.0002	Normal		
- Subset (3022)	Zenith angle encoder gae,2 [gon]	0.0002	Normal		
- Subset (4010)	Zenith angle encoder obE,1 [gon]	0.0002	Normal		
- Subset (4012)	Zenith angle encoder obE,2 [gon]	0.0002	Normal		=
- Subset (4013)	Tilting axis error σα [gon]	0.0003	Normal		
- Subset (4014)	Collimation error oy [gon]	0.0003	Normal		
- Subset (4015)	Axis offset σ <sub>ex</sub> [m]	0.000005	Normal		
- Subset (4016)	Beam offset oby [m]	0.000005	Normal		
- 🗋 Subset (4017)	Beam offset σ <sub>bz</sub> [m]	0.000005	Normal		
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– 🗋 Subset (3053) 📃	Settings Polar Observations (a-priori)	Transformation Pa	arameters	Local.	<u></u> ) 🛛
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Fig. 3: Stochastic Model Setup: Uncertainties and Related Distribution Functions.

Fig. 1: Local Tie Survey at Onsala Space Observatory with tilted Laser Tracker LTD840.

ted to the gravity field like laser trackers. In this case, each instrument station defines an independent local coordinate system (see e.g. Fig. 1). These local systems are combined via concatenated spatial similarity transformations. To combine the raw observations (slop distance *d*, yaw angle  $\Theta$  and pitch angle  $\Phi$ ) on The use of spatial similarity transformations paves a simple way to provide local observation in a global context like the ITRF. Whereas in the conventional approach the ITRF-transformation process is carried out as a final step, in our approach the transformation into the global reference frame takes place right in the beginning of the bundle adjustment.

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