Coordinate Based Bundle Adjustment

Advanced Network Adjustment Model for Polar Measurement Systems

Abstract

To fulfil the GGOS requirements on local-ties at co-location stations, measurements with high precision instruments and data analysis with rigorous uncertainty propagation are necessary. To evaluate the results of e.g. high performance total stations or laser trackers, the accuracy-limiting parameters of the measurement process have to be quantified and projected onto an uncertainty model. Using the generally and interdisciplinarily accepted *Guide to the Expression of Uncertainty in Measurement* (GUM) a transparent and traceable stochastic model can be derived. A Cartesian coordinate based bundle adjustment is suggested to integrate the local measurements into a global context, avoiding gravitational influences. The included comprehensive uncertainty model is based on a specific geometric model of a polar measurement system and takes instrument specific and target dependent error parameters into account.

Concept of Rigorous Bundle Adjustment

The network adjustment is the first and most important processing and conditioning step. As a result, the network adjustment provides the spatial coordinates and their corresponding uncertainties as full variance-covariance matrix. Depending on the extent of the local network and the accuracy requirements, the influence of the curvature of the earth cannot be neglected.

In metrology, coordinate based algorithms are developed, because most of the instruments are unrela-

Fig. 1: Local Tie Survey at Onsala Space Observatory with tilted Laser Tracker LTD840.

ted to the gravity field like laser trackers. In this case, each instrument station defines an independent local coordinate system (see e.g. Fig. 1). These local systems are combined via concatenated spatial similarity transformations. To combine the raw observations (slop distance d, yaw angle Θ and pitch angle Φ) on the level of Cartesian coordinates, the well known conversion has to be applied.

$$\mathbf{p}_{i,j}(\Theta, \Phi, d) = \begin{pmatrix} x \\ y \\ z \end{pmatrix}_{i,j} = \begin{pmatrix} d \sin\Phi \cos\Theta \\ d \sin\Phi \sin\Theta \\ d \cos\Phi \end{pmatrix}_{i,j} \quad (1)$$

Deriving Uncertainties of Observations

The use of Equation 1 assumes a perfect instrument which means, that:

- the azimuth and elevation axes are orthogonal to each other and have an intersection point,
- the distance measurement unit is centred w.r.t. this intersection point,
- the target beam is coaxial and orthogonal to the elevation axis,
- the normal vector of the angle encoder is aligned and centred to the rotation axis, and
- the angle encoder is free of graduation errors.

For manufacturing reasons, deviations from the idealized model can appear.

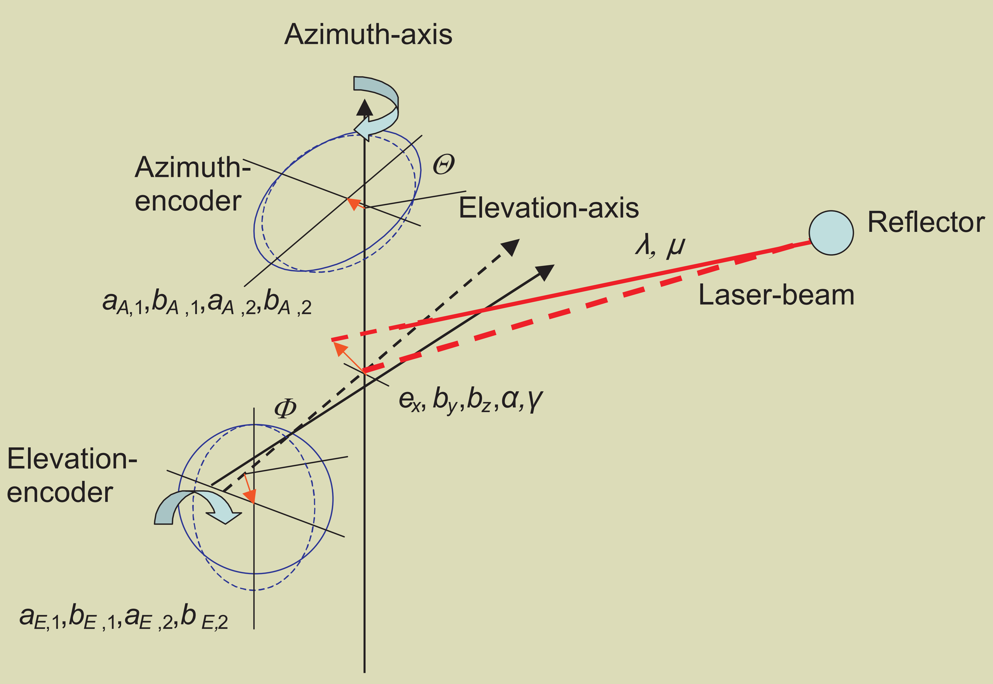

Fig. 2: Irregularities of a Polar Measurement Instrument (Hughes et al., 2011).

Hughes et al. (2011) formulated a 16 parameter comprehensive compensation model for a laser tracker and suggested a network configuration for deriving these calibration parameters (see Fig. 2). Whereas the calibration parameters are used to derive corrected Cartesian coordinates, the uncertainties of the calibration parameters and their related distributions have to be introduced to the stochastic model of the least-squares adjustment. The resulting correlations are also taken into account. Beside the propagation of uncertainty, Monte-Carlo simulation based techniques are proposed by the *Guide to the Expression of Uncertainty in Measurement* (GUM).

Table 1: Calibration Parameters of a Polar Measurement Instrument.

Type	Description	
λ	Displacement offset	
μ	Scaling parameter	
e_x	Axis offset	
b_y	Y-laser offset	
b_z	Z-laser offset	
α	Trunnion axis error	
γ	Horizontal collimation error	
$a_{(A,1)}$	Azimuth-encoder error,	1th Fourier coefficient
$b_{(A,1)}$		1th Fourier coefficient
$a_{(A,2)}$		2nd Fourier coefficient
$b_{(A,2)}$		2nd Fourier coefficient
$a_{(E,0)}$	Vertical collimation error	
$a_{(E,1)}$	Elevation-encoder error,	1th Fourier coefficient
$b_{(E,1)}$		1th Fourier coefficient
$a_{(E,2)}$		2nd Fourier coefficient
$b_{(E,2)}$		2nd Fourier coefficient

Developed Prototype Software

At the *Frankfurt Laboratory for Industrial Metrology* a prototype software has been developed to integrate local measurements into a global context (Lösler & Eschelbach 2012). Based on the compensation model of Hughes et al. (2011) an extended stochastic model with 25 parameters is implemented (Fig. 3).

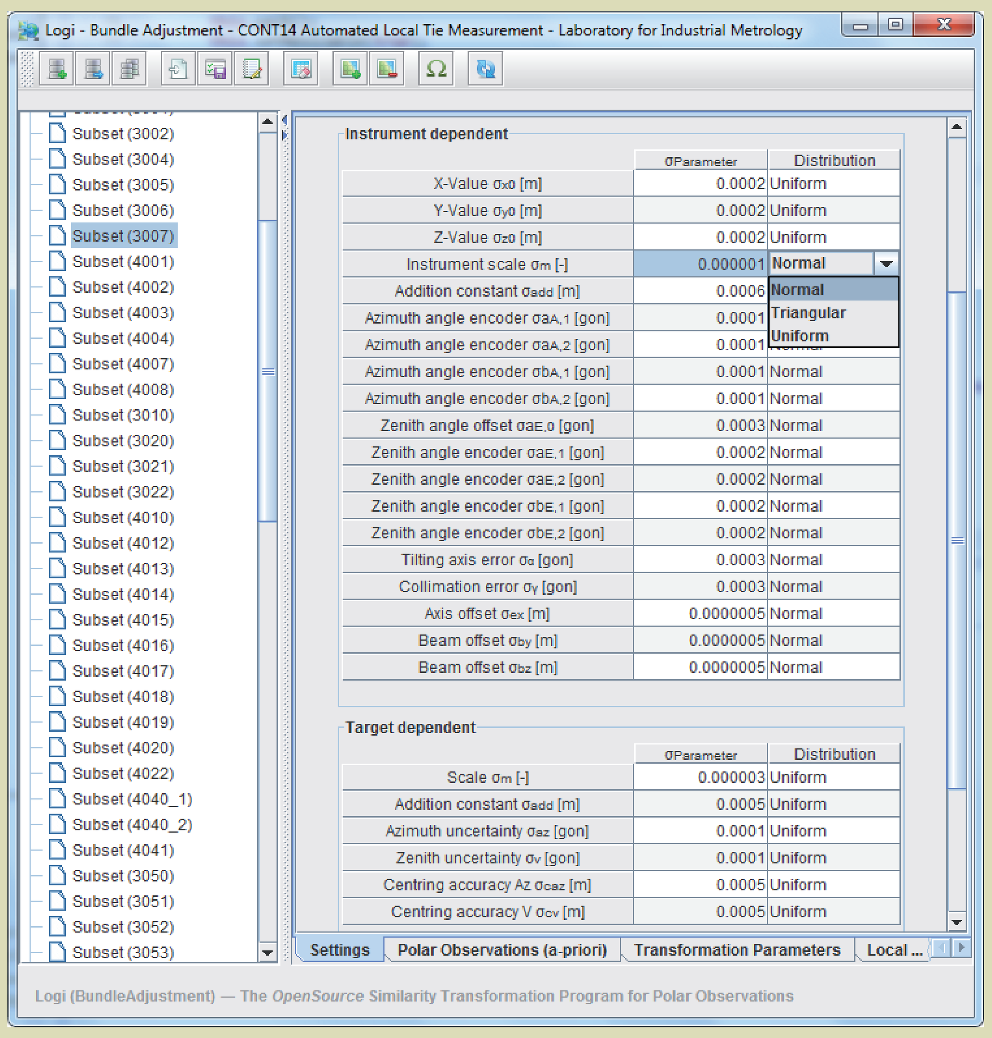

Fig. 3: Stochastic Model Setup: Uncertainties and Related Distribution Functions.

The use of spatial similarity transformations paves a simple way to provide local observation in a global context like the ITRF. Whereas in the conventional approach the ITRF-transformation process is carried out as a final step, in our approach the transformation into the global reference frame takes place right in the beginning of the bundle adjustment.

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